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Visual Sensitivities to Color Differences in Daylight*

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An apparatus is described which facilitates the presentation of pairs of variable colors without variation of luminance. With this instrument, various criteria of visual sensitivity to color difference have been investigated. The standard deviation of color matching was finally adopted as the most reproducible criterion. The test field was two degrees in diameter, divided by a vertical biprism edge, and was viewed centrally with a surrounding field of forty-two degrees diameter uniformly illuminated so as to have a chromaticity similar to that of the I.C.I. Standard Illuminant *C* (average daylight). The luminance of the test field was maintained constant at 15 millilamberts, and the surrounding field was 7.5 millilamberts. These fields were viewed monocularly through an artificial pupil, 2.6 mm in diameter. Over twenty-five thousand trials at color matching have been recorded for a single observer, and the readings are analyzed in detail and compared with previously available data. The standard deviations of the trials are represented in terms of distance in the standard 1931 I.C.I. chromaticity diagram. These increments of distance are represented as functions of position along straight lines in the chromaticity diagram, and also as functions of direction of departure from points representing certain standard chromaticities. Such representations are simpler than the traditional representations of wave-length thresholds and purity thresholds as functions of wave-length, and the accuracy of the representations is improved by this simplicity. Chromaticity discrimination for non-spectral colors is represented simultaneously and on the same basis as for spectral colors. Small, equally noticeable chromaticity differences are represented for all chromaticities and for all kinds of variations by the lengths of the radii of a family of ellipses drawn on the standard chromaticity diagram. These ellipses cannot be transformed into equal-sized circles by any projective transformation of the standard chromaticity diagram. The consistency of these data with the results of other investigators is exhibited in terms of the noticeabilities of wave-length differences in the spectrum and of the noticeabilities of purity differences from a neutral stimulus, as functions of dominant wave-length.

THE measurement and specification of color is a well-established technique. Familiarity with the fundamentals of this subject will be assumed throughout this report.[1] Color specifications reveal whether or not two sources radiating different spectral distributions appear to have the same color, for the average human observer. Similarly, the colors of reflecting materials are specified in a manner which indicates immediately whether or not several samples having different spectral reflectance characteristics appear alike under certain conditions of illumination. These specifications give precise meaning to color standards. The specifications of color standards can be recorded, and can be communicated, in the form of quantitative values which are reproducible at will in all adequately equipped laboratories. Color standards need no longer be dependent upon the preservation of material samples of questionable permanence.

* Communication No. 840 from the Kodak Research Laboratories.

[1] A. C. Hardy and collaborators, *Handbook of Colorimetry* (Technology Press, Cambridge, Massachusetts, 1936).

The standard specifications of color also provide a useful system for the classification of colors. The luminance[2] of a source of radiant energy, or of an object reflecting radiant energy, is the most familiar of the quantities in terms of which colors may be specified and classified. Alternatively, luminous reflectance may be used to specify the ratio between the luminance of an object and the luminance of a perfect diffuse reflector similarly illuminated. This ratio is characteristic of the reflecting surface, and consequently luminous reflectance is a convenient specification when the color of an object is to be discussed without reference to the illuminance[2] incident upon it.

The chromaticity, or quality of a color may be specified and classified in terms of dominant wave-length and purity.[1] Alternatively, the chromaticity of a color may be specified by a point in a plane diagram. This diagram represents the relative chromaticities of all colors in much the same manner as a plane map represents the relative locations of various places on the earth. An example of the representation of chromaticities by points on the chromaticity diagram is given in Fig. 1. As in the case of a plane map of an imperfectly spherical world, the relations exhibited between locations in this diagram are limited in significance. In a certain limited sense, differences of specifications correspond in order of magnitude to the apparent differences of colors. For example, if two adjacent objects having the same dominant wave-length and luminance differ in purity, the object having the greater purity will appear more saturated. An estimate of the ratio of the apparent saturations may not confirm the ratio of the measured purities, but equality, or the fact of greater or less purity will be confirmed subjectively. Similarly, two adjacent objects having the same luminance and purity but differing in dominant wave-length will differ in the subjectively appreciated quality called hue, in the same sense as they differ in dominant wave-length. A third object sharing the common values of luminance and purity but having a dominant wave-length greater or less than both of the original colors will exhibit a hue more nearly resembling that of the object from which it differs less with respect to dominant wave-length. The visually estimated ratio of differences in hue will not, in general, equal the ratio of the corresponding differences of dominant wave-length, but an object having a dominant wave-length intermediate between the dominant wave-lengths of two other objects will appear intermediate in hue also. In the presence of differences of luminance or purity, or both, equality of dominant wave-length does not necessarily assure equality of hue. Similarly, in the presence of differences in luminance or dominant wave-length, equality of purity rarely corresponds to equality of saturation. Finally, most of the difficulties peculiar to heterochromatic photometry arise from the fact that in the presence of differences in dominant wave-length or purity, or both, comparisons of brightness are difficult and unsatisfactory.

Many investigations of the noticeability of color differences[3–5] have been undertaken for the purpose of interpreting measured differences

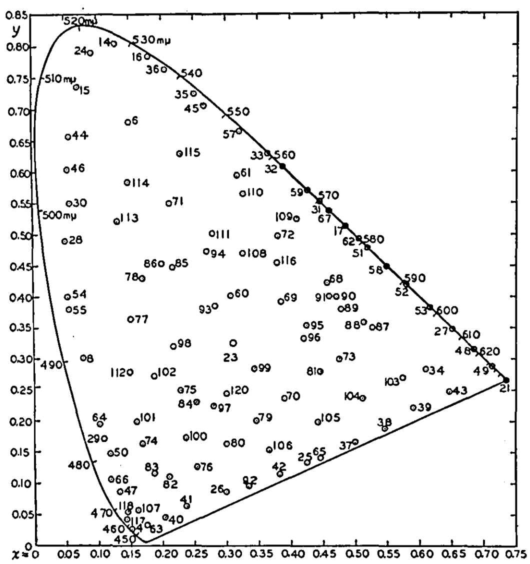


FIG. 1. Chromaticities of filters used in discrimination apparatus.

[2] L. A. Jones, "Colorimetry: Preliminary draft of a report on nomenclature and definitions," J. Opt. Soc. Am. **27**, 207–213 (1937).

[3] D. B. Judd, "A Maxwell triangle yielding uniform chromaticity scales," J. Opt. Soc. Am. **25**, 24–35 (1935).

[4] I. G. Priest and F. G. Brickwedde, "Minimum perceptible colorimetric purity as a function of dominant wave-length," J. Opt. Soc. Am. **28**, 133–139 (1938).

[5] W. D. Wright, "The sensitivity of the eye to small colour differences," Proc. Phys. Soc. **53**, 93–112 (1941).

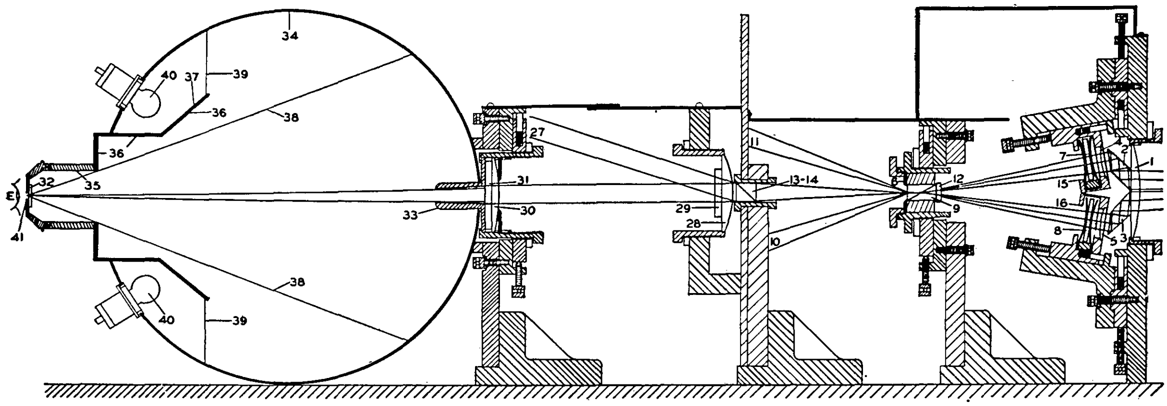


FIG. 2. Vertical cross section of chromaticity discrimination apparatus.

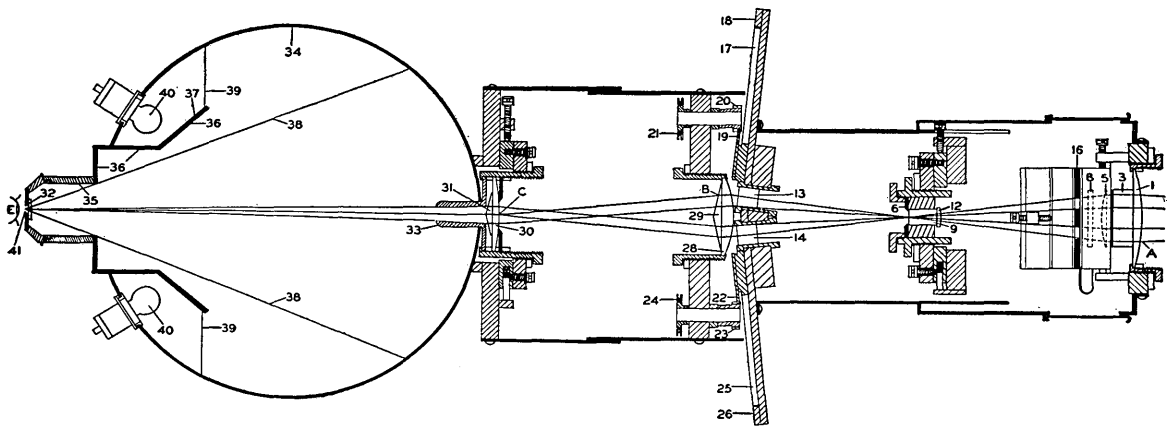


FIG. 3. Horizontal cross section of chromaticity discrimination apparatus.

in terms comparable with subjective estimates. In the present report, the results of such an investigation will be reported in terms of the standard 1931 I.C.I. specifications of the colors. No modification of this standard system of specifying colors will be recommended. Any modification of the chromaticity diagram adequately representing these data on noticeabilities of color differences would be so complicated as to be useless in practice. Such a sacrifice of the simplicity of the standard system is unnecessary, in view of the ease with which the supplementary data concerning noticeability can be applied to color specifications expressed in the standard system.

A. APPARATUS

The apparatus employed in this investigation was designed to provide the following features:

1. A two-degree comparison field, divided semicircularly.
2. A surrounding field of twenty-one degrees radius, of uniform color, which may be changed or extinguished, to provide any reasonable conditions of adaptation.
3. The observer is called upon to manipulate only a single control in order to produce the desired variations of color in each half of the comparison field.
4. Any selected kind of variation of chromaticity from any initial chromaticity can be produced in either half of the field, with (a) automatically constant luminance, if desired, or (b) the luminance can be made to vary proportionately in any desired ratio with any selected kind of chromaticity variation, or (c) the luminance alone of any selected chromaticity can be varied.

5. All visual stimuli are produced by additive mixtures of filtered beams from a single calibrated light source. The resultant heterogeneous stimuli are more nearly similar to ordinary visual stimuli than are the mixtures of homogeneous spectral stimuli frequently used for visual research. The results therefore correspond more closely than usual to the most practically important circumstances of vision. The stimuli produced in this apparatus are specified accurately by standard colorimetric analyses of the filters employed, and errors which frequently arise from unsuspected impurities of spectral components are avoided.

6. When the two halves of the comparison field are matched visually, their energy distributions are identical, so that individual differences of color-mixture characteristics have a minimum effect on the data for chromaticity sensibility. Errors of calibration of the color filters result in only unimportant second-order errors in the color differences calculated from those calibrations.

These features will be made evident by the description of the construction and functioning of the apparatus and by the discussion of its calibration and the method of investigation.

A vertical cross section of the apparatus is shown in Fig. 2. A horizontal cross section is shown in Fig. 3. The vertical cross section follows the axial rays of one of the two pairs of light paths, such as the ray indicated by *A-B-C-E* in Fig. 3. The light source, which is situated to the right of the instrument as represented in the drawings, is not shown on the drawings. It consists of a ground-glass plate illuminated by the light from a 6-volt projection lamp (Mazda C-8, vertical coil filament, clear bulb, T-10, medium prefocus base). This lamp is operated at the current which has been found by previous calibration to be necessary to maintain the desired 2848°K color temperature of the light entering the instrument from the illuminated ground glass. A ribbon filament lamp could be used in place of the coil filament and ground-glass combination if much higher field illuminance were desired. The ground-glass source has been used exclusively in the experiments which are now being reported. All observations were made with 15 millilamberts field luminance.

The light from each point of the extended source is collimated by the lens *1*. The light passing through the upper half of this lens is reflected vertically upwards and then downwards at an angle of ten degrees from the horizontal by two total internal reflections in the prism *2*. The other half of the collimated beam is reflected downwards and then upwards at an angle of ten degrees from the horizontal by two total internal reflections in prism *3*. The light in these two separated, but converging beams is then focused by lenses *4* and *5* to produce two coincident images of the light source in the plane of the diaphragm *6*. Light filters can be placed in the separated beams at *7* and *8*. Each of the two beams which intersect within the Wollaston double-image polarizing prism *9* is split into two beams, each diverging ten degrees in the vertical plane from the direction of the incident beam. Thus, the beam from the upper filter *7* is split into two beams, one of which proceeds along the horizontal axis of the instrument, and the other of which is deflected downwards at an angle of twenty degrees from the horizontal, being ultimately intercepted and absorbed by the blackened plate at *10*. The portion of the light from the upper incident beam which is directed along the axis by the Wollaston prism is polarized in the vertical plane. The beam from the lower filter *8* is also split by the Wollaston prism into two beams, one of which proceeds along the horizontal axis, together with part of the light from the upper incident beam. The other of the two beams into which the lower incident beam is split is deflected upwards at an angle of twenty degrees to the axis of the instrument and is intercepted and absorbed by the blackened plate at *11*. The portion of the light from the lower incident beam which is directed along the axis by the Wollaston prism is polarized in the horizontal plane. Coincident images of the light filters at *7* and *8* are focused by the lens *12* onto the pair of Rochon prisms at *13* and *14*.

The diaphragm *15* intercepts the light from all portions of the upper filter *7* except two areas which are focused by lens *12* onto the Rochon prisms *13* and *14*. Similarly, the diaphragm *16* intercepts the light from all portions of the lower filter *8* except two areas which are focused by lens *12* onto the Rochon prisms *13* and *14*.

Each of the Rochon prisms is mounted so that it can be rotated at will within a range of 90 degrees. Prism *13* is rigidly connected to the pointer *17* which indicates the azimuth of the prism on the scale *18* which is graduated in degrees. A vernier on the pointer *17* makes possible the recording of tenths of degrees. Prism *13* is rotated by the attached toothed segment *19* which meshes with the gear *20*. This gear *20* is activated by the chain drive *21* which is geared to a control knob which may be manipulated by the observer. Prism *14* is rotated by the gears *22* and *23* and chain *24*, coupling to a second control knob. The azimuth of prism *14* is indicated by a vernier on the pointer *25* read against the divided scale *26*.

The proportions of the beams from the filters *7* and *8* which are transmitted undeviated by each Rochon prism depend upon the azimuth of the prism. The portions of the beams which are deviated by the Rochon prisms are intercepted and absorbed by the blackened plate, as indicated at *27*. The scale readings are 0 degrees when the Rochon prisms transmit, without deviation, light polarized in a vertical plane, and 90 degrees when the undeviated beams are polarized in a horizontal plane. Consequently, if U represents the energy distribution of the light from the upper filter *7* incident on one of the Rochon prisms, and if V represents the energy distribution of the light from the lower filter *8* incident upon the same prism, then the energy distribution of the light transmitted undeviated by the prism is

$$W=T(U\sin^2\theta+V\cos^2\theta), \quad (1)$$

where θ is the azimuth of the prism, as indicated by the scale reading, and T is the transmittance of the Rochon prism for light of such polarization that it is transmitted without deviation. If the values of any additive functions of the distributions U and V are equal, then the value of the same function of the transmitted distribution is constant, independently of all variations of θ and consequently of the proportions in which U and V are combined. In particular, if the filters at *7* and *8* have equal luminous transmittances for the I.C.I. Illuminant A (2848°K), which is the quality of the light incident upon them, then the luminous flux transmitted without deviation by each of the Rochon prisms will be constant, independently of θ and consequently of the proportions in which the colors from the two filters *7* and *8* are combined after transmission by the Rochon prisms. This property of the apparatus is responsible for the feature (4*a*) of automatically constant field luminance for the additive mixtures in all proportions of different colors, such as the colors of U and V. Similarly, if the luminous transmittances of the filters at *7* and *8* are unequal, the variation of field luminance will be proportional to the variation of chromaticity resulting from the variation of the proportions of the two beams transmitted undeviated by the Rochon prisms. In this manner the experimental condition (4*b*) can be secured. Finally, of course, the condition (4*c*) can be realized by substituting an opaque plate for the filter at *8* thus converting the instrument into one type of polarization photometer.

The beams transmitted without deviation by the Rochon prisms are collimated by the lens *28*. The biprism *29* refracts these beams and causes their axial rays to intersect on the dihedral edge of the field biprism *31*. The field lens *30* focuses coincident images of the light source on the pupil of the observer's eye E. The dihedral edge of the biprism *31* is at the focal point of the eye lens *32* so that the eye lens of the normal observer is completely relaxed. Other eye lenses can be substituted for this lens *32* to compensate for variations from normal refraction in the eyes of various observers. The boundary of the test field is defined by the end of the cylindrical field stop *33*. The inner bore of this cylinder is sufficiently conical so that none of the inner surface can be seen by the observer at E. The circular edge of this conical hole, nearest the eye, serves as the field stop. The hidden interior surface of this tube is coated with a dull black finish to absorb most of the light refracted from the axial beams by the field biprism. This useless light is kept at a minimum by the diaphragms *15* and *16* which are of such shape as to intercept all except those portions of the filtered beams which are necessary to illuminate the test field. The length of the tube *33* is sufficient to shield the biprism *31* from all parts of the whitened interior of the sphere *34*. The surfaces of the biprism *31* and the field lens *30* are coated with a "non-reflecting" layer which further reduces the possibility of reflected stray light in the test field. This treatment of the surfaces of the biprisms *29* and *31* and of the lenses *28* and *30* also reduces to negligible magnitudes the variation of reflection losses at these surfaces, resulting from the oblique incidence of the polarized beams from the rotatable prisms *13* and *14*. The interiors of the eye tube *35* and of the lamp shield *36* are finished dull black, to minimize the reflection of light from the convex surface of the field lens *30*. The biprism *31* is cemented to the plane surface of the field lens *30* to eliminate the reflection of light from these plane surfaces. Similarly, the biprism *29* is cemented to the plane surface of the lens *28*. The external surfaces of the field stop *33*, the entire inner surface of the sphere *34*, and the outer surface *37* of the lamp shield are painted white. An opaque layer of smoke from burning magnesium ribbon was deposited simultaneously on the cylindrical field stop *33* and on the hemisphere visible from E, in order to obliterate completely all joints and irregularities. The illumination of the interior of the sphere is so uniform and diffused that the outlines of the cylindrical field stop *33* can be distinguished only with effort. The radius of the illuminated surrounding field is limited by the length of the eye tube *35*. The limiting rays are shown by *38*. The angular extent of the illuminated surrounding field could be increased somewhat by shortening the eye tube *35* or by increasing its diameter, but the advantage would appear to be negligible in comparison with the increased difficulties of construction, and discomfort of the observer, who would be required to place his head very close to the sphere lamps and their connections. An annular ring *39* of filter film can be placed between the lamp shield and the sphere, to introduce whatever chromaticity may be desired in the surrounding field. The intensity of the surrounding field can be changed by altering the number or size of the lamps *40* employed, or by varying the current passing through them. The chromaticity and the luminance of the surrounding field are calibrated by establishing a color match between the surrounding field and the test field. The colorimetric specifications of the surrounding field are then equal to those calculated for the test field from the azimuth of the Rochon prisms and the colorimetric specifications of the filters, placed at *7* and *8*. The various adjusting and clamping devices which are provided for the focusing and centering of the optical elements are indicated in Figs. 2 and 3. The appropriate procedure for placing such an instrument in adjustment can be devised by any person familiar with the maintenance of precision optical instruments. An artificial pupil may be placed over the eye lens at *41*. Such an artificial pupil, with a diameter of 2.6 millimeters, was used to fix the level of retinal illumination in the experiments described in this paper.

This instrument is designed for monocular observation but it could conceivably be converted into a binocular instrument. Equal transmissions for both eyes, and constant transmission for all planes of polarization are essential requirements of any binocular viewing device employed with this instrument. The Maxwellian field, and the large illuminated surrounding field impose further requirements which eliminate practically all known binocular eyepieces. Color-mixture data, luminosity data, and sensibilities to color differences are occasionally, although not commonly, distinctly different for the right and left eye of a single observer. Such differences are not confined to observers who exhibit definitely anomalous color discrimination, but have also been detected in the case of observers classed as "normal" by all of the usual tests. Such differences would result in erratic results of binocular observations. The results of binocular observations would not be expected to average consistently the different characteristics of the two eyes. Rather, one eye would be expected to dominate and govern the results of the majority of the observations, with the subordinate eye gaining dominance occasionally and contributing erratic observations, as the result of either conscious or unconscious binocular rivalry.

A movable eye shield which permits the use of either eye has been employed throughout the monocular observations which are the subject of this report. This shield protects the idle eye from all light, permitting the idle eye to remain open and relaxed.

TABLE I. Filters for chromaticity instrument (in Illuminant A).

Filter number	Constituents Chromatic	Constituents Neutral	X	Y	Z	m	x	y
4	34,38	0.07	0.01640	0.00280	0.08855	0.10775	0.152	0.026
6	65,8	1.25	0.00059	0.00265	0.00066	0.00390	0.150	0.680
8	75	0.42	0.00071	0.00280	0.00575	0.00911	0.077	0.302
14	X1	0.24	0.00044	0.00276	0.00023	0.00343	0.128	0.805
15	3(63)		0.00027	0.00280	0.00076	0.00383	0.070	0.736
16	3(74)	0.10	0.00062	0.00270	0.00011	0.00353	0.180	0.785
17	2(73)	0.50	0.00256	0.00272	0.00000	0.00528	0.485	0.514
21	70	0.50	0.00769	0.00280	0.00000	0.01049	0.737	0.263
23	78	1.17	0.00278	0.00294	0.00339	0.00911	0.305	0.323
24	X2	0.10	0.00035	0.00297	0.00043	0.00375	0.092	0.790
25	35	0.51	0.00943	0.00295	0.00980	0.02218	0.425	0.133
26	X3	0.65	0.01153	0.00311	0.02400	0.03864	0.299	0.086
27	72	0.78	0.00549	0.00293	0.00000	0.00842	0.651	0.348
28	4(65A)		0.00029	0.00286	0.00268	0.00583	0.050	0.490
29	45A	0.55	0.00188	0.00298	0.01247	0.01733	0.108	0.172
30	3(65)	0.08	0.00029	0.00278	0.00199	0.00506	0.057	0.550
31	21,53	1.21	0.00216	0.00268	0.00000	0.00484	0.446	0.553
32	16,53	1.45	0.00181	0.00283	0.00002	0.00466	0.388	0.608
33	2(16),2(53)	0.48	0.00151	0.00260	0.00002	0.00413	0.365	0.630
34	31,86C	1.84	0.00583	0.00270	0.00102	0.00955	0.610	0.283
35	2(61),15	1.00	0.00097	0.00280	0.00009	0.00386	0.251	0.725
36	2(61),12	1.12	0.00075	0.00280	0.00011	0.00366	0.205	0.764
37	35,86C	0.68	0.00919	0.00305	0.00611	0.01835	0.500	0.166
38	35,86B	0.66	0.00879	0.00302	0.00427	0.01608	0.546	0.188
39	35,86A	0.70	0.00784	0.00294	0.00252	0.01330	0.590	0.221
40	34,78A	0.30	0.01293	0.00275	0.04780	0.06348	0.203	0.045
41	34,78B	0.45	0.01075	0.00285	0.03180	0.04540	0.237	0.063
42	35,78B	0.40	0.00987	0.00297	0.01304	0.02588	0.382	0.115
43	35,86A	0.60	0.00756	0.00290	0.00125	0.01171	0.646	0.248
44	63,2(65)		0.00028	0.00284	0.00132	0.00444	0.057	0.656
45	61,12	1.67	0.00105	0.00280	0.00011	0.00396	0.266	0.706
46	3(65),2(4)		0.00025	0.00278	0.00161	0.00464	0.054	0.603
47	49A	0.47	0.00603	0.00301	0.03600	0.04504	0.133	0.067
48	25	2.00	0.00605	0.00280	0.00000	0.00885	0.684	0.316
49	29	1.71	0.00695	0.00280	0.00000	0.00975	0.712	0.288
50	46	0.95	0.00225	0.00283	0.01405	0.01913	0.118	0.148
51	15	2.60	0.00355	0.00290	0.00001	0.00646	0.550	0.449
52	21	2.45	0.00382	0.00278	0.00000	0.00660	0.580	0.420
53	22	2.35	0.00453	0.00280	0.00000	0.00733	0.617	0.382
54	75,65A,2(4)		0.00038	0.00290	0.00396	0.00724	0.053	0.400
55	75,65A,4		0.00040	0.00275	0.00408	0.00723	0.055	0.380
57	61,2(15)	1.46	0.00136	0.00280	0.00005	0.00421	0.322	0.666
58	2(16)	2.50	0.00378	0.00285	0.00001	0.00664	0.570	0.429
59	21,53	0.90	0.00209	0.00280	0.00001	0.00490	0.427	0.571
60	38,78B	1.92	0.00206	0.00270	0.00195	0.00671	0.307	0.402
61	59	2.15	0.00149	0.00280	0.00040	0.00470	0.318	0.595
62	53,22	0.93	0.00287	0.00280	0.00000	0.00567	0.506	0.494
63	34,78AA		0.01450	0.00275	0.06550	0.08275	0.175	0.033
64	75,45	0.56	0.00177	0.00310	0.01296	0.01783	0.102	0.195
65	35,2	0.49	0.00850	0.00270	0.00783	0.01903	0.446	0.1415
66	2(46)	0.36	0.00320	0.00285	0.02083	0.02688	0.119	0.106
67	2(21),53	1.00	0.00230	0.00269	0.00000	0.00499	0.460	0.539
68		2.60	0.00296	0.00274	0.00078	0.00648	0.457	0.423
69	78A	2.13	0.00257	0.00262	0.00149	0.00668	0.385	0.392
70	78,30	0.85	0.00478	0.00290	0.00457	0.01225	0.390	0.237
71	69	1.87	0.00110	0.00285	0.00123	0.00518	0.212	0.550
72	66	2.37	0.00217	0.00285	0.00070	0.00572	0.380	0.498
73	78AA,30	1.34	0.00460	0.00290	0.00217	0.00967	0.475	0.300
74	78,78AA	0.65	0.00265	0.00259	0.01052	0.01576	0.168	0.164
75	78,78B	1.15	0.00251	0.00275	0.00574	0.01100	0.228	0.250
76	78,78AA,30	0.25	0.00522	0.00260	0.01280	0.02062	0.253	0.126
77	44A	1.62	0.00110	0.00280	0.00354	0.00729	0.151	0.364
78	44	1.70	0.00111	0.00250	0.00261	0.00652	0.170	0.430
79	78,78B,30	0.60	0.00503	0.00290	0.00656	0.01449	0.347	0.200
80	78,78A,30	0.45	0.00570	0.00310	0.01020	0.01900	0.300	0.163
81	78AA,78C,30	1.23	0.00421	0.00265	0.00260	0.00946	0.445	0.280
82	78,78AA,X4	0.37	0.00550	0.00288	0.01780	0.02618	0.210	0.110
83	78,78AA,X5	0.45	0.00438	0.00272	0.01633	0.02343	0.187	0.116
84	78,78AA,X6	0.60	0.00307	0.00280	0.00630	0.01217	0.252	0.230
85	67	1.70	0.00149	0.00308	0.00231	0.00688	0.217	0.448
86	67,78C	1.60	0.00132	0.00300	0.00229	0.00661	0.200	0.454
87	30	2.20	0.00445	0.00295	0.00104	0.00844	0.527	0.350
88	X4	2.30	0.00416	0.00290	0.00105	0.00811	0.513	0.358
89	X7	2.44	0.00387	0.00307	0.00115	0.00809	0.478	0.380
90	X8	2.44	0.00330	0.00280	0.00090	0.00700	0.471	0.400
91	X6	2.48	0.00327	0.00285	0.00099	0.00711	0.460	0.401
92	X9,34	0.70	0.01055	0.00300	0.01803	0.03158	0.334	0.095
93	38,78B	1.85	0.00194	0.00265	0.00229	0.00688	0.282	0.385
94	51,38A	1.77	0.00175	0.00306	0.00166	0.00647	0.270	0.473
95	X7,78A	1.77	0.00361	0.00300	0.00188	0.00849	0.425	0.354
96	X5,78A	1.74	0.00378	0.00300	0.00223	0.00901	0.420	0.333
97	X7,78	1.74	0.00387	0.00312	0.00687	0.01386	0.279	0.225
98	38A,78B	1.50	0.00190	0.00280	0.00405	0.00875	0.217	0.320
99	X7,78AA	1.45	0.00380	0.00315	0.00413	0.01108	0.343	0.284
100	X7,78,78A	0.84	0.00385	0.00281	0.00957	0.01623	0.237	0.173
101	38A,78	0.80	0.00224	0.00280	0.00895	0.01399	0.160	0.200
102	78A,38A	1.20	0.00195	0.00285	0.00564	0.01044	0.187	0.273
103	31	1.87	0.00627	0.00295	0.00171	0.01093	0.574	0.270
104	31,78B	1.50	0.00669	0.00310	0.00330	0.01309	0.511	0.237
105	31,78A	1.16	0.00622	0.00280	0.00505	0.01407	0.442	0.199
106	31,78AA	0.73	0.00694	0.00290	0.00912	0.01896	0.366	0.153
107	X5,2(78)	0.10	0.00763	0.00270	0.03700	0.04733	0.161	0.057
108	66,78A	1.70	0.00201	0.00290	0.00126	0.00617	0.326	0 470
109	59A	1.28	0.00242	0.00310	0.00038	0.00590	0.410	0.525
110	59A,38	0.95	0.00182	0.00315	0.00060	0.00557	0.327	0.565
111	59A,78	1.13	0.00175	0.00315	0.00138	0.00628	0.279	0.502
112	3(38A)	1.13	0.00141	0.00265	0.00542	0.00948	0.149	0.279
113	65	1.28	0.00065	0.00260	0.00174	0.00499	0.131	0.521
114	65,86	0.88	0.00069	0.00270	0.00125	0.00464	0.148	0.583
115	60	1.84	0.00103	0.00280	0.00063	0.00446	0.230	0.629
116	2(51),78A	1.75	0.00213	0.00255	0.00092	0.00560	0.380	0.455
117	48A	0.32	0.00944	0.00280	0.05312	0.06536	0.144	0.042
118	48A,2(2A)	0.24	0.00766	0.00285	0.04235	0.05276	0.145	0.054
119	34,30	0.72	0.00928	0.00290	0.00976	0.02194	0.423	0.132
120	X7,78AA,78B	1.20	0.00369	0.00300	0.00561	0.01230	0.300	0.244

B. CALIBRATED COLOR FILTERS

A set of about 100 color filters has been prepared for use with this instrument. The luminous transmittances of these filters are all within 10 percent of their average, 0.00285. The luminous transmittances of these filters, and their chromaticities, have been calculated from their spectral transmittances and for the standard I.C.I. observer and Illuminant A. The filters consist of one or more pieces of gelatin filter film cemented between microscope slide glasses. Most of the filter components were selected from the commercial Wratten filters. A few special filter films were required in order to secure chromaticities unobtainable with combinations of the standard filters. These special filter films were designed and made by Mr. Albert Smith, of these Laboratories, whose interest and assistance is here gratefully acknowledged.

Table I indicates the constituents and colorimetric specifications of the assembled filters. The chromaticities of these filters are represented in Fig. 1. The serial numbers with which these filters are designated in Fig. 1 and column 1 of Table I bear no relation to the Wratten filter numbers of the components. The serial numbers correspond approximately to the order in which the filters were assembled. The Wratten filter numbers of the constituents of each filter are listed in the second column. Special filter components are numbered serially, with the letter X preceding each of these numbers to avoid confusion with the Wratten filter numbers. In many cases, two or more pieces of one type of filter film were used. Such cases are indicated by parentheses around the Wratten filter number preceded by a numeral indicating the number of pieces used. In the fourth column is given the density of the Wratten neutral filter film em-

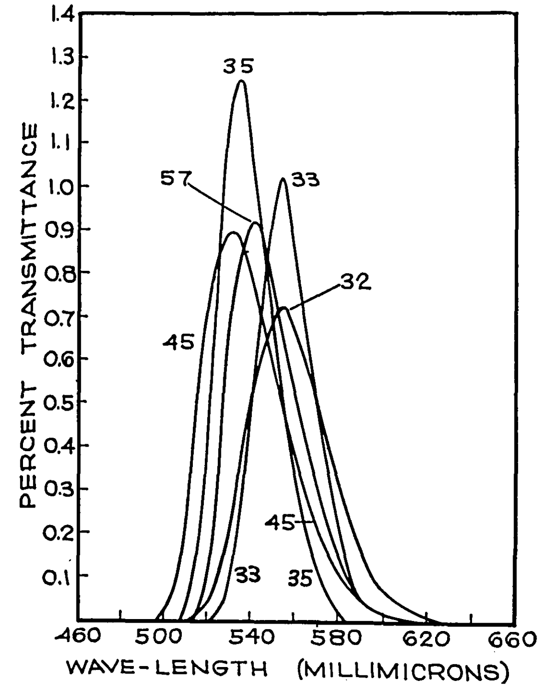


FIG. 4. Spectrophotometric curves of five filters.

ployed to adjust the various filters to the desired value of luminous transmittance. The I.C.I. tristimulus values, X, Y, and Z are given in the fifth, sixth, and seventh columns. The value of Y is the luminous transmittance of each filter. The sum m of the three tristimulus values is given in the eighth column. This value is the weight which may be assigned to the color transmitted by each filter in computations of the chromaticities of mixtures according to the center of gravity principle.[6,7] The coordinates, x, y, of the points representing each filter in the chromaticity diagram, Fig. 1, are given in columns 9 and 10 of Table I.

These filters have such low luminous transmittances that in most cases sufficiently accurate spectrophotometric data could not be secured by direct measurements of the finished filters. The spectrophotometric data, such as those represented by the curves shown in Figs. 4 and 5, have been determined in various ways, but mostly by spectrophotometric measurement of the separate components of each filter, followed by computation of the transmittance of the cemented combinations. The effects of cementing on total absorption and in the reduction of reflection losses were carefully investigated and taken into account in these computations. As a final check, the relative luminous transmittances of filters of similar chromaticities were compared visually. The instrument represented in Figs. 2 and 3 was used for these comparisons. This apparatus functioned as a polarization photometer, when an opaque plate was placed at position *8*. The two filters being compared were placed at *7*, each filter covering one of the openings in the diaphragm *15*. Filters represented by neighboring points in Fig. 1 have sufficiently similar chromaticities so that these heterochromatic comparisons of luminous transmittances could be made quite satisfactorily. The relative luminous transmittances of filters compared in this manner are inversely proportional to the ratio of the squares of the sines of the azimuths of the corresponding Rochon prisms. This use of the sine square law is very favorable for the comparison of nearly equal transmittances, such as are involved in the comparisons of the set of filters employed here. Systematic variations from the computed values of the luminous transmittances were observed and correlated with the positions of the points representing the filters in the chromaticity diagram. These systematic variations were confirmed by repetitions of the comparisons by the same observer on different days, but were different for different observers. Although these variations and correlations require further study in relation to the actual luminosity curves for the observers involved, before any positive conclusions can be stated, these variations are tentatively attributed to variations of the luminosity curves of the observers from the standard I.C.I. data used in the computations. In particular, all observers seemed to have higher than standard luminosity values for the extreme red end of the spectrum,

[6] D. L. MacAdam, "The theory of the maximum visual efficiency of colored materials," J. Opt. Soc. Am. **25**, 249–252 (1935).

[7] D. L. MacAdam, "Projective transformations of I.C.I. color specifications," J. Opt. Soc. Am. **27**, 294–299 (1937).

and somewhat lower than standard luminosity values for the extreme violet end of the spectrum. Such an effect might be expected and attributed to the differences in the conditions of observation from those employed by Gibson and Tyndall.[8,9] In particular, the large illuminated surrounding field, and the maintenance of a fairly high constant retinal illuminance (200 photons, or 8 lux) for all comparisons, including the extreme red, violet, and purples, may have contributed appreciably to this effect. Although the high, central portions of the standard curve, which are usually most important in practice, were determined at normal levels of retinal illuminance,[8] the relative luminosities of wave-lengths in the extreme red and violet regions of the spectrum were determined with considerably lower retinal illuminances. Consequently, near the ends of the visible spectrum the rates of change of the standard curve with wave-length may be more nearly representative of the scotopic curve than of the photopic curve. Such an effect would not cause appreciable errors in most practical problems, but would be expected to give rise to noticeable discrepancies for filters whose luminous transmittances occur largely at either or both of the extremes of the visible spectrum. No attempt was made to compensate for the systematic discrepancies between the observed and computed luminous transmittances. Very few discrepancies, other than the systematic variations, were detected between the observed and computed values. The few isolated errors were traced to spectrophotometric or computational errors. Such errors were corrected and eliminated before Table I and Figs. 1, 4, and 5 were completed.

All of the filters were stored so as to minimize changes in any of the components which may be unstable. The use of unstable components was avoided as far as possible, and those filters which contained constituents suspected of instability were checked periodically, both spectrophotometrically and visually.

[8] K. S. Gibson and E. P. T. Tyndall, "Visibility of radiant energy," Sci. Papers Nat. Bur. Standards 19, 131–191 (1924); Trans. Illum. Eng. Soc. 19, 176–196 (1924).

[9] *Proceedings of the Sixth Session, International Commission on Illumination* (Geneva, 1924), pp. 67 and 232.

C. COLORIMETRIC CALIBRATION OF APPARATUS

Colorimetric specifications of the mixtures obtained in the test field of the instrument represented in Figs. 2 and 3 can be computed by use of the tristimulus values of the filters in equations similar to Eq. (1). Let the symbols X_u, Y_u, Z_u represent the tristimulus values of the field when $\theta=90$ degrees, that is, when the color of the field is produced by the filter at position *7*, alone. Let the symbols X_v, Y_v, Z_v represent the tristimulus values of the field when $\theta=0$ degrees, that is, when the color of the field is produced by the filter at position *8*, alone. Then, for any scale reading θ the tristimulus values X, Y, Z of the mixture are given by the equations:

$$X=X_u \sin^2\theta+X_v \cos^2\theta, \quad (2)$$

$$Y=Y_u \sin^2\theta+Y_v \cos^2\theta, \quad (3)$$

$$Z=Z_u \sin^2\theta+Z_v \cos^2\theta. \quad (4)$$

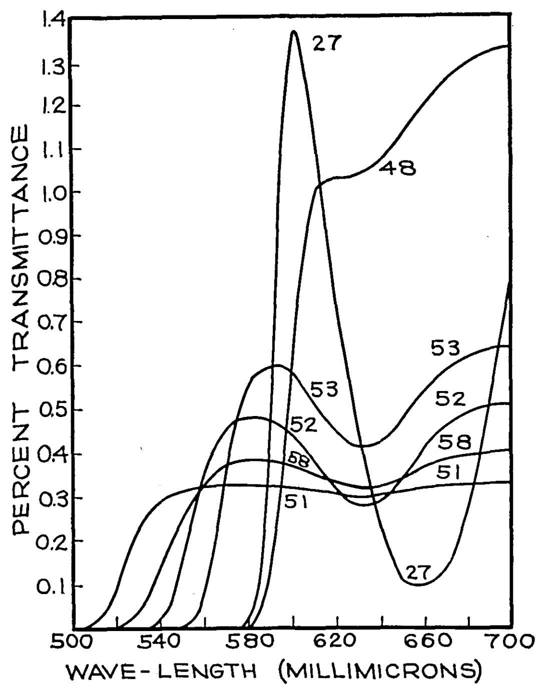


FIG. 5. Spectrophotometric curves of six filters.

Since the value of Y is the luminance of the field, the luminance of any mixture is given by Eq. (3). If the luminous transmittances of the filters at *7* and *8* are equal for the quality of light which is incident upon them (Illuminant A), then the luminance Y of the field is constant for all values of θ.

The coordinates of the point representing the mixture in the chromaticity diagram can be calculated from the values X, Y, Z given by Eqs. (2), (3), and (4). The center of gravity principle provides a much more convenient method of locating the point. Let the symbol m_u represent the "mass" $X_u+Y_u+Z_u$, assigned in Table I to the filter at position *7*, which produces the color of the field when $\theta=90$ degrees. Let the symbol m_v represent the "mass" $X_v+Y_v+Z_v$ assigned in Table I to the filter at position *8*. Then the point representing the mixture for the scale reading θ divides the straight line joining the points representing the filters in the ratio $m_u \sin^2\theta$ to $m_v \cos^2\theta$. Let the chromaticity of the field when $\theta=90$ degrees be represented by the point U. Let the chromaticity of the field when $\theta=0$ degrees be represented by the point V. Let the chromaticity of the field for the scale reading θ be represented by the point W. If the ratio of the distances VW/VU is represented by the fraction f then

$$f=1/(r\cot^2\theta+1), \quad (5)$$

where

$$r=m_v/m_u.$$

The fractional distance of W from V toward U may be estimated for any values of r and θ from

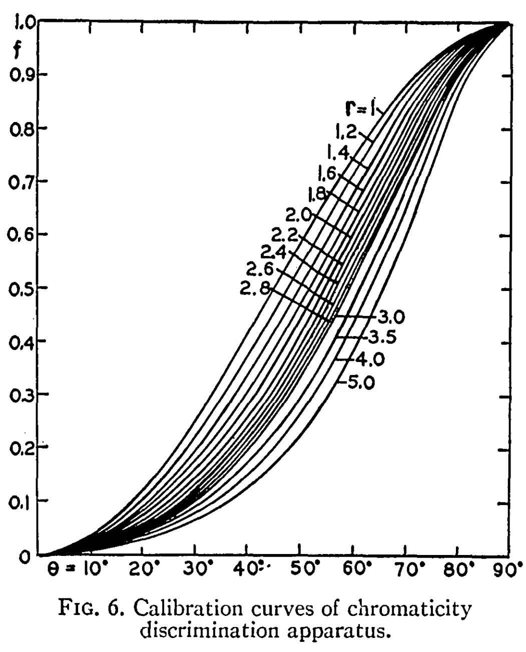


FIG. 6. Calibration curves of chromaticity discrimination apparatus.

the curves shown in Fig. 6. A special slide rule was designed and used for all of the computations of f involved in this investigation. This slide rule, illustrated in Fig. 7, avoids the difficulties and errors of interpolation encountered in the use of Fig. 6. The bottom scale A on the frame of the rule represents values of the ratio r from 0.01 to 100.0. The distances in inches to the right from the center index to the divisions are given by the equation:

$$D=5\log_{10} r. \quad (6)$$

The lower scale B on the slide represents reciprocals of the ratio r from $1/r=0.004$ to $1/r=40.0$. Scale B is divided in exactly the same manner as is scale A, except that the index is displaced two inches to the right of the center of the slide. The upper scale T on the slide represents values of the angle θ from 9 degrees to 86.5 degrees. The distances in inches to the right from the index to the divisions of this scale are given by the equation:

$$D=10\log_{10}\cot\theta. \quad (7)$$

The 45-degree index is at exactly the same position on the slide as the unit index on scale B, that is, two inches to the right of the center of the slide. Finally, the top scale F on the frame, represents values of the fraction f from 0.01 to 0.99. The distances in inches from the center index to the divisions of this scale are given by the equation:

$$D=5\log_{10}(1-f)/f. \quad (8)$$

In all of the scales, positive values of D are measured to the right of the index and negative values to the left. When the 45-degree index of the slide is aligned with the value of r on the A scale, the value of f can be found on the F scale opposite the value of θ on the T scale. Alternatively, the value of the reciprocal of the ratio $1/r$ on the B scale can be aligned with the center index of the frame. Since Eq. (5) reduces to $f=\sin^2\theta$, when $r=1.0$, the values of $\sin^2\theta$ required in Eq. (3) for the calculation of luminance can be determined by setting the 45-degree index of the slide at $r=1.0$ and reading the values of $\sin^2\theta$ from the F scale. The values of $\cos^2\theta$ which are also required in Eq. (3) are easily determined by subtracting the values of $\sin^2\theta$ from 1.0.

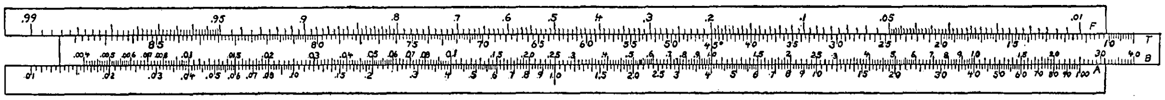


FIG. 7. Slide rule for calculation of chromaticities synthesized in discrimination apparatus.

The distance VW of the point representing the mixture from the point representing the chromaticity of the field when $\theta=0$ degrees is equal to the value f multiplied by the distance UV between the points representing the chromaticities of the field when $\theta=0$ degrees and $\theta=90$ degrees. When the distance UV is measured in the same units as the coordinate scales of the chromaticity diagram, it is represented by the symbol S.

$$S=[(x_u-x_v)^2+(y_u-y_v)^2]^{\frac{1}{2}}, \tag{9}$$

where
$$x_u=X_u/m_u, \quad y_u=Y_u/m_u,$$
$$x_v=X_v/m_v, \quad y_v=Y_v/m_v.$$

The distance VW measured in the same units as the coordinate scales is represented by the symbol s. The distances S and s may be measured with reasonable accuracy by use of a pair of dividers with a chromaticity diagram drawn accurately to a fairly large scale. The multiplication,

$$s=fS, \tag{10}$$

can be carried out with comparable accuracy by using in the customary manner the A and B scales of the slide rule represented in Fig. 7.

Differences of the chromaticities corresponding to small changes, $\Delta\theta$, of the position of either Rochon prism *13* or *14* in Fig. 3, can be determined with great accuracy by calculating the product

$$\Delta s=\Delta\theta(ds/d\theta). \tag{11}$$

A sufficiently accurate value of the derivative can be found by subtracting the values of f indicated by the slide rule for neighboring values of θ, dividing this difference by the difference of the values of θ, and multiplying this ratio by the value of S. Small differences of chromaticity computed from Eq. (11) are considerably more accurate than the chromaticities of the filters used to synthesize the colors.

D. STANDARD DEVIATIONS OF COLOR MATCHING

Several criteria were investigated for the measurement of the noticeability of color differences. The just noticeable difference was used extensively with results consistent with those which are given in this paper, but intercomparisons of data for different series of colors were unsatisfactory because of rather great fluctuations to which this criterion appeared to be subject. A consistent system of data, including representative kinds of color difference throughout the entire gamut of colors attainable with the instrument, could not be determined satisfactorily with a criterion subject to such fluctuations. Adjustments of the data to compensate for such fluctuations were considered inadmissible, since they might be influenced by preconceptions concerning the probable trend of the data and accumulated errors of such adjustments might introduce spurious characteristics and obscure essential features of the results.

Attempts were made to adjust color differences to equal certain standard color differences, several times greater than just noticeable. Observations of the standard color differences were alternated with the adjustments of the test differences. This criterion provided more consistent results than the criterion of just noticeability, but was exceedingly tedious and required extensive training of the observer. Such training cannot be given to subjects who are available for only a short examination during which an adequate determination of color discrimination must be completed. These examinations are one of the most important immediate applications of the instrument employed in the present investigation, and a criterion was sought which would be applicable in both routine testing and research. Such a criterion must be more reliable than the just noticeable difference and require less training and experience than either that or the judgment of equally noticeable color differences.

König and Dieterici[10] employed the mean error

[10] A. König and C. Dieterici, "Über die Empfindlichkeit des normalen Auges für Wellenlangenunterschiede des Lichtes," Wied. Ann. d. Physik und Chemie **22**, 579–589 (1884); Graefes Archiv. **30** (2), 158 (1884); *Gesammelte Abhandlungen zur Physiologischen Optik* (Leipzig, 1903), p. 23.

TABLE II. Standard deviations of color matching.

0° Filter number	90° Filter number	θ° Average setting	Δθ° Standard deviation	Average chromaticity x	y	Distance from 0° point =s	Δs Standard deviation	Δy/Δx
4	23	83.8	0.239	0.286	0.286	0.294	0.00271	1.94
4	23	79.1	0.256	0.259	0.233	0.232	0.00370	1.94
4	23	79.1	0.230	0.259	0.233	0.232	0.00332	1.94
4	23	74.1	0.247	0.230	0.177	0.171	0.00290	1.94
4	23	69.0	0.382	0.208	0.135	0.122	0.00312	1.94
4	23	68.5	0.304	0.206	0.131	0.117	0.00242	1.94
4	23	58.4	0.335	0.180	0.081	0.062	0.00150	1.94
4	23	42.1	0.477	0.162	0.045	0.022	0.00067	1.94
4	23	25.1	0.563	0.155	0.031	0.006	0.00028	1.94
100	23	43.7	0.823	0.259	0.254	0.056	0.00213	2.21
66	23	73.9	0.284	0.268	0.280	0.229	0.00163	1.16
66	23	60.9	0.314	0.216	0.219	0.139	0.00184	1.16
66	23	52.0	0.334	0.185	0.183	0.102	0.00156	1.16
66	23	47.2	0.430	0.169	0.165	0.080	0.00176	1.16
66	23	45.1	0.423	0.166	0.161	0.072	0.00154	1.16
66	23	32.8	0.507	0.142	0.133	0.035	0.00116	1.16
66	23	16.7	0.642	0.125	0.113	0.008	0.00064	1.16
64	23	73.0	0.400	0.274	0.303	0.203	0.00157	0.63
64	23	58.7	0.302	0.220	0.269	0.139	0.00139	0.63
64	23	44.3	0.372	0.169	0.237	0.078	0.00139	0.63
64	23	30.2	0.652	0.132	0.214	0.035	0.00156	0.63
64	23	15.9	0.988	0.110	0.200	0.009	0.00122	0.63
8	23	74.1	0.531	0.288	0.322	0.213	0.00113	0.09
8	23	59.6	0.376	0.247	0.318	0.172	0.00147	0.09
8	23	45.2	0.504	0.192	0.313	0.116	0.00206	0.09
8	23	30.9	0.591	0.137	0.308	0.061	0.00215	0.09
23	55	21.7	0.469	0.277	0.329	0.029	0.00119	−0.23
23	55	30.7	0.322	0.251	0.335	0.056	0.00112	−0.23
23	55	31.1	0.326	0.249	0.336	0.057	0.00117	−0.23
23	55	45.3	0.296	0.193	0.349	0.116	0.00131	−0.23
23	55	46.0	0.430	0.191	0.349	0.118	0.00170	−0.23
23	55	46.2	0.436	0.190	0.349	0.119	0.00172	−0.23
23	55	60.6	0.614	0.126	0.364	0.185	0.00265	−0.23
23	55	61.2	0.587	0.124	0.364	0.187	0.00245	−0.23
23	55	75.6	1.080	0.074	0.376	0.238	0.00278	−0.23
23	55	77.4	0.953	0.070	0.377	0.243	0.00211	−0.23
23	28	21.4	0.432	0.282	0.338	0.027	0.00111	−0.66
23	28	30.7	0.430	0.258	0.354	0.057	0.00142	−0.66
23	28	45.2	0.332	0.205	0.389	0.120	0.00173	−0.66
23	28	45.7	0.391	0.203	0.390	0.123	0.00203	−0.66
23	28	59.6	0.490	0.139	0.432	0.200	0.00275	−0.66
23	28	74.0	0.778	0.079	0.471	0.271	0.00310	−0.66
23	46	20.2	0.370	0.289	0.341	0.024	0.00094	−1.12
23	46	20.8	0.515	0.288	0.342	0.026	0.00125	−1.12
23	46	30.1	0.425	0.269	0.364	0.055	0.00162	−1.12
23	46	30.4	0.298	0.278	0.365	0.056	0.00113	−1.12
23	46	30.8	0.327	0.267	0.366	0.058	0.00123	−1.12
23	46	44.6	0.461	0.222	0.416	0.126	0.00232	−1.12
23	46	45.8	0.441	0.218	0.420	0.132	0.00259	−1.12
23	46	45.5	0.410	0.219	0.419	0.130	0.00241	−1.12
23	46	58.4	0.423	0.162	0.483	0.218	0.00308	−1.12
23	46	59.4	0.437	0.157	0.489	0.225	0.00320	−1.12
23	46	59.9	0.536	0.153	0.491	0.228	0.00393	−1.12
23	46	60.9	0.510	0.149	0.497	0.235	0.00374	−1.12
23	46	59.8	0.428	0.154	0.491	0.228	0.00311	−1.12
23	46	70.8	0.690	0.102	0.549	0.305	0.00498	−1.12
23	46	74.1	0.928	0.088	0.564	0.327	0.00527	−1.12
23	46	77.5	1.027	0.076	0.578	0.346	0.00507	−1.12
38	23	31.8	0.572	0.503	0.212	0.050	0.00182	−0.56
38	23	46.0	0.376	0.456	0.248	0.104	0.00171	−0.56
38	23	60.6	0.267	0.391	0.275	0.179	0.00141	−0.56
38	23	75.8	0.232	0.329	0.309	0.250	0.00090	−0.56
65	23	30.7	0.392	0.426	0.168	0.033	0.00089	−1.29
65	23	31.1	0.408	0.426	0.168	0.033	0.00093	−1.29
65	23	45.3	0.250	0.400	0.201	0.075	0.00089	−1.29
65	23	75.4	0.172	0.323	0.300	0.201	0.00059	−1.29
65	23	75.4	0.187	0.323	0.300	0.201	0.00065	−1.29
25	23	31.9	0.460	0.408	0.159	0.031	0.00093	−1.58
25	23	45.4	0.363	0.390	0.189	0.066	0.00119	−1.58
25	23	60.9	0.234	0.357	0.240	0.127	0.00107	−1.58
25	23	76.2	0.187	0.320	0.298	0.196	0.00071	−1.58
42	23	15.2	0.865	0.380	0.120	0.006	0.00065	−2.70
42	23	21.3	0.584	0.378	0.126	0.011	0.00065	−2.70
42	23	31.1	0.372	0.373	0.138	0.025	0.00065	−2.70
42	23	44.5	0.250	0.362	0.168	0.059	0.00077	−2.70
42	23	60.4	0.221	0.342	0.223	0.116	0.00102	−2.70
42	23	74.4	0.241	0.319	0.285	0.182	0.00099	−2.70
79	23	28.9	0.614	0.340	0.220	0.021	0.00087	−2.92
79	23	44.0	0.408	0.332	0.245	0.048	0.00084	−2.92
79	23	73.9	0.603	0.310	0.309	0.115	0.00106	−2.92
41	23	30.2	0.477	0.241	0.080	0.017	0.00059	3.82
41	23	44.9	0.427	0.248	0.106	0.045	0.00113	3.82
41	23	45.1	0.421	0.249	0.107	0.046	0.00111	3.82
41	23	60.3	0.296	0.263	0.163	0.103	0.00151	3.82
76	23	29.6	0.706	0.260	0.142	0.026	0.00127	3.80
76	23	33.5	0.729	0.262	0.158	0.033	0.00155	3.80
76	23	44.4	0.637	0.268	0.163	0.061	0.00178	3.80
76	23	60.8	0.563	0.288	0.241	0.121	0.00231	3.80
76	23	74.9	0.555	0.298	0.295	0.176	0.00176	3.80
40	23	30.1	0.662	0.208	0.058	0.014	0.00069	2.73
40	23	45.1	0.531	0.216	0.080	0.038	0.00122	2.73
40	23	60.0	0.388	0.234	0.128	0.090	0.00193	2.73
40	23	74.7	0.293	0.270	0.228	0.195	0.00274	2.73
63	23	15.2	1.503	0.176	0.035	0.003	0.00058	2.23
63	23	28.7	1.090	0.179	0.043	0.010	0.00078	2.23
63	23	33.6	1.345	0.181	0.046	0.013	0.00127	2.23
63	23	40.0	1.010	0.184	0.054	0.021	0.00133	2.23
63	23	41.6	1.082	0.185	0.056	0.026	0.00173	2.23
63	23	45.2	0.810	0.188	0.062	0.029	0.00153	2.23
63	23	50.0	0.920	0.193	0.082	0.039	0.00226	2.23
63	23	59.4	0.759	0.206	0.103	0.076	0.00384	2.23
63	23	59.6	0.800	0.207	0.103	0.078	0.00405	2.23
63	23	69.3	0.443	0.232	0.160	0.140	0.00370	2.23
63	23	70.1	0.443	0.234	0.166	0.148	0.00370	2.23
63	23	74.7	0.354	0.252	0.205	0.190	0.00373	2.23
63	23	79.0	0.296	0.272	0.250	0.238	0.00350	2.23
23	24	21.9	0.422	0.292	0.352	0.032	0.00130	−2.19
23	24	31.2	0.458	0.277	0.384	0.068	0.00214	−2.19
23	24	44.3	0.508	0.245	0.454	0.145	0.00378	−2.19
23	24	59.4	0.551	0.190	0.576	0.280	0.00575	−2.19
23	24	73.1	0.880	0.131	0.704	0.422	0.00857	−2.19
23	36	30.0	0.539	0.293	0.375	0.054	0.00206	−4.41
23	36	44.5	0.470	0.277	0.445	0.126	0.00307	−4.41
23	36	58.2	0.526	0.254	0.549	0.233	0.00470	−4.41
23	36	72.3	0.794	0.225	0.675	0.363	0.00697	−4.41
23	36	72.9	0.777	0.224	0.680	0.369	0.00679	−4.41
23	33	28.6	1.043	0.312	0.360	0.038	0.00284	5.03
23	33	44.2	0.777	0.323	0.415	0.094	0.00350	5.03
23	33	59.1	0.812	0.338	0.494	0.176	0.00496	5.03
23	33	74.4	1.385	0.356	0.585	0.268	0.00729	5.03
23	59	15.2	1.585	0.310	0.332	0.011	0.00219	2.04
23	59	18.7	1.438	0.312	0.338	0.016	0.00293	2.04
23	59	25.8	1.000	0.319	0.351	0.031	0.00246	2.04
23	59	41.6	0.689	0.341	0.397	0.084	0.00286	2.04
23	59	42.5	0.681	0.341	0.400	0.088	0.00286	2.04
23	59	58.0	0.774	0.376	0.466	0.162	0.00418	2.04
23	59	69.6	1.037	0.402	0.520	0.223	0.00563	2.04
23	17	22.8	0.866	0.322	0.341	0.024	0.00185	1.06
23	17	44.7	0.510	0.370	0.392	0.096	0.00215	1.06
23	17	59.9	0.608	0.419	0.444	0.168	0.00299	1.06
23	17	75.9	1.028	0.468	0.496	0.239	0.00368	1.06
23	51	17.4	0.557	0.321	0.331	0.018	0.00116	0.51
23	51	29.1	0.360	0.349	0.346	0.051	0.00122	0.51
23	51	43.8	0.397	0.402	0.373	0.110	0.00190	0.51
23	51	43.8	0.378	0.402	0.373	0.110	0.00181	0.51
23	51	59.1	0.437	0.468	0.407	0.186	0.00219	0.51
23	51	72.4	0.835	0.520	0.434	0.244	0.00302	0.51
23	53	13.3	0.415	0.318	0.326	0.014	0.00087	0.19
23	53	28.9	0.246	0.365	0.334	0.062	0.00102	0.19
23	53	43.8	0.334	0.437	0.348	0.136	0.00179	0.19
23	53	58.4	0.452	0.518	0.363	0.218	0.00243	0.19
23	53	71.3	0.668	0.578	0.375	0.280	0.00268	0.19
34	23	75.6	0.271	0.325	0.320	0.291	0.00073	−0.13
34	23	60.5	0.218	0.381	0.313	0.233	0.00104	−0.13
34	23	46.5	0.267	0.453	0.304	0.160	0.00147	−0.13
34	23	41.5	0.289	0.479	0.300	0.134	0.00154	−0.13
34	23	32.1	0.390	0.527	0.294	0.085	0.00187	−0.13
34	23	23.3	0.462	0.564	0.289	0.046	0.00176	−0.13
21	23	75.5	0.166	0.336	0.319	0.405	0.00070	−0.14
21	23	60.4	0.210	0.422	0.307	0.318	0.00150	−0.14
21	34	35.2	1.342	0.698	0.269	0.039	0.00262	−0.16

TABLE II.—Continued.

0° Filter number	90° Filter number	θ° Average setting	Δθ° Standard deviation	Average chromaticity x	y	Distance from 0° point =s	Δs Standard deviation	Δy/Δx
4	40	10.2	0.663	0.153	0.026	0.001	0.000135	0.37
4	40	20.3	0.379	0.156	0.027	0.004	0.000156	0.37
4	40	30.1	0.296	0.160	0.029	0.009	0.000179	0.37
4	40	39.8	0.266	0.167	0.032	0.016	0.000218	0.37
4	40	45.2	0.254	0.171	0.033	0.020	0.000224	0.37
4	40	50.1	0.274	0.175	0.035	0.025	0.000268	0.37
4	40	60.0	0.360	0.185	0.038	0.035	0.000372	0.37
4	40	69.7	0.532	0.194	0.042	0.044	0.000483	0.37
4	40	70.2	0.509	0.194	0.042	0.045	0.000463	0.37
4	66	21.0	1.733	0.151	0.031	0.003	0.00068	−2.43
4	66	29.5	1.185	0.150	0.032	0.006	0.00057	−2.43
4	66	29.9	1.297	0.150	0.032	0.006	0.00062	−2.43
4	66	45.1	1.100	0.145	0.042	0.017	0.00104	−2.43
4	66	68.5	0.860	0.132	0.075	0.053	0.00178	−2.43
4	66	69.0	1.060	0.131	0.077	0.054	0.00219	−2.43
66	55	34.2	1.280	0.112	0.136	0.031	0.00274	−4.28
66	55	44.1	0.715	0.106	0.161	0.057	0.00222	−4.28
66	55	60.1	0.470	0.090	0.229	0.127	0.00265	−4.28
66	55	60.2	0.536	0.090	0.230	0.127	0.00302	−4.28
66	55	70.1	0.422	0.076	0.291	0.190	0.00286	−4.28
66	55	70.2	0.400	0.076	0.292	0.190	0.00271	−4.28
66	55	74.1	0.560	0.070	0.317	0.217	0.00363	−4.28
66	55	80.1	0.770	0.062	0.352	0.253	0.00393	−4.28
66	55	81.6	1.000	0.060	0.360	0.261	0.00450	−4.28
55	15	29.3	1.053	0.057	0.431	0.052	0.00385	16.18
55	15	43.8	0.747	0.060	0.499	0.118	0.00419	16.18
55	15	59.3	0.595	0.064	0.594	0.215	0.00409	16.18
55	15	59.4	0.790	0.064	0.595	0.216	0.00542	16.18
55	15	59.5	0.806	0.064	0.595	0.217	0.00554	16.18
55	15	73.7	0.985	0.068	0.687	0.308	0.00530	16.18
55	15	74.3	1.046	0.068	0.690	0.312	0.00563	16.18
55	15	74.4	1.012	0.068	0.691	0.312	0.00545	16.18
15	16	27.5	1.264	0.092	0.746	0.024	0.00205	0.45
15	16	28.1	1.441	0.093	0.746	0.025	0.00234	0.45
15	16	42.9	1.092	0.118	0.758	0.054	0.00225	0.45
15	16	59.3	1.009	0.150	0.772	0.088	0.00196	0.45
15	16	60.3	1.237	0.151	0.772	0.090	0.00240	0.45
17	16	20.9	0.445	0.465	0.532	0.036	0.00156	−0.89
17	16	30.2	0.339	0.436	0.558	0.076	0.00170	−0.89
17	16	30.4	0.370	0.436	0.558	0.078	0.00185	−0.89
17	16	30.6	0.397	0.435	0.559	0.078	0.00198	−0.89
17	16	30.8	0.419	0.434	0.560	0.079	0.00210	−0.89
17	16	30.7	0.327	0.435	0.559	0.079	0.00164	−0.89
17	16	40.4	0.353	0.392	0.596	0.135	0.00222	−0.89
17	16	45.0	0.308	0.370	0.616	0.165	0.00214	−0.89
17	16	45.4	0.348	0.368	0.618	0.167	0.00237	−0.89
17	16	45.6	0.327	0.367	0.619	0.169	0.00222	−0.89
17	16	50.1	0.357	0.343	0.641	0.201	0.00258	−0.89
17	16	59.2	0.352	0.291	0.687	0.268	0.00261	−0.89
17	16	59.7	0.418	0.283	0.694	0.272	0.00309	−0.89
17	16	60.1	0.486	0.281	0.696	0.275	0.00360	−0.89
17	16	60.5	0.456	0.278	0.698	0.278	0.00338	−0.89
17	16	73.1	0.815	0.217	0.752	0.360	0.00418	−0.89
17	16	74.9	0.738	0.210	0.758	0.370	0.00378	−0.89
17	16	75.3	0.890	0.208	0.760	0.372	0.00455	−0.89
21	33	21.6	0.397	0.723	0.276	0.019	0.00074	−0.99
21	33	31.2	0.358	0.706	0.293	0.043	0.00114	−0.99
21	33	46.2	0.216	0.659	0.340	0.109	0.00127	−0.99
21	33	50.7	0.188	0.638	0.360	0.138	0.00137	−0.99
21	33	60.5	0.136	0.576	0.422	0.225	0.00141	−0.99
21	33	70.4	0.112	0.492	0.505	0.343	0.00143	−0.99
21	33	75.3	0.143	0.447	0.549	0.406	0.00179	−0.99
48	33	21.6	0.521	0.661	0.338	0.032	0.00154	−0.98
48	33	45.6	0.161	0.576	0.422	0.153	0.00115	−0.98
48	33	60.5	0.178	0.491	0.506	0.272	0.00155	−0.98
48	33	70.3	0.179	0.430	0.566	0.358	0.00147	−0.98
40	21	29.3	0.354	0.229	0.056	0.028	0.00078	+0.41
40	21	39.4	0.282	0.256	0.067	0.058	0.00107	0.41
40	21	40.2	0.299	0.260	0.068	0.061	0.00114	0.41
40	21	49.9	0.369	0.304	0.086	0.109	0.00240	0.41
40	21	50.2	0.326	0.306	0.087	0.111	0.00212	0.41
40	21	60.1	0.366	0.381	0.118	0.192	0.00379	0.41
40	21	69.9	0.287	0.499	0.166	0.320	0.00447	0.41
40	21	74.5	0.268	0.566	0.193	0.392	0.00456	0.41
40	21	75.0	0.268	0.575	0.197	0.402	0.00456	0.41
40	21	79.6	0.308	0.646	0.226	0.478	0.00499	0.41
40	21	79.8	0.324	0.648	0.227	0.479	0.00524	0.41
40	21	84.6	0.498	0.705	0.240	0.545	0.00502	0.41
40	21	84.7	0.514	0.710	0.241	0.548	0.00518	0.41

of adjustment in their investigation of the noticeability of wave-length differences in the spectrum. This method has been mentioned as a possibility by several other investigators, but it has not been used to any considerable extent. In a very significant recent report[5] of an investigation similar to the present, Wright discussed his reasons for adopting a criterion of definitely noticeable color differences. He concludes this discussion with the following paragraph:

"It is, of course, quite true that some variations in judgment are inevitable. No doubt the ideal method would be to make a large number of matches at each point in the colour chart and then to analyse the spread of the observations, but in practice this would be an impossibly lengthy process."

The program which Wright describes as ideal becomes practical with the apparatus used in the present investigation, in which any one desired variable can be investigated independently of all other variations. In an oral report[11] of the present investigation, the results were presented in terms of the probable errors of color matching. In the discussion of that report, the suggestion was made that the results be published in terms of the standard deviation, the root mean square of the individual deviations from the average setting. The significance of the concept of probable error is questionable when the distribution of errors does not fit the "normal" distribution curve very accurately. In order to avoid implications not justified by the observations, the results will be reported here in terms of standard deviation rather than in terms of probable error. This change in the data reported previously[11] involved only the omission of a constant factor, 0.6745.

The apparatus employed in this investigation is particularly suitable for the matching method

[11] D. L. MacAdam, "Noticeability of color differences in daylight," J. Opt. Soc. Am. **30**, 657A (1940).

TABLE III. Standard deviations of color matching.

0° Filter number	90° Filter number	$\theta°$ Average setting	$\Delta\theta°$ Standard deviation	Average chromaticity x	y	$\Delta y/\Delta x$	Δs Standard deviation
80	16	63.1	0.281	0.258	0.450	−5.18	0.00388
84	35	57.1	0.445	0.258	0.450	−500	0.00424
23	14	44.1	0.586	0.258	0.450	−2.72	0.00413
64	33	65.0	0.292	0.258	0.450	1.66	0.00323
90	28	48.8	0.241	0.258	0.450	−0.47	0.00188
90	28	49.1	0.240	0.258	0.450	−0.47	0.00186
74	57	66.5	0.320	0.258	0.450	3.26	0.00398
23	14	43.8	0.563	0.258	0.450	−2.72	0.00397
60	44	32.4	0.548	0.258	0.450	−1.02	0.00237
54	17	48.0	0.304	0.258	0.450	0.26	0.00234
54	17	48.1	0.317	0.258	0.450	0.26	0.00244
80	36	64.0	0.315	0.258	0.450	−6.33	0.00430
106	104	52.7	0.874	0.441	0.198	0.58	0.00270
119	73	50.7	0.400	0.441	0.198	3.24	0.00123
25	81	51.0	0.452	0.441	0.198	7.35	0.00108
79	38	39.7	0.352	0.441	0.198	−0.06	0.00137
79	38	40.0	0.306	0.441	0.198	−0.06	0.00119
79	38	39.6	0.387	0.441	0.198	−0.06	0.00150
80	43	44.4	0.300	0.441	0.198	0.25	0.00190
42	52	53.5	0.350	0.441	0.198	1.54	0.00176
92	53	52.8	0.338	0.441	0.198	1.02	0.00241
65	70	50.0	0.401	0.441	0.198	−1.70	0.00095
37	120	44.5	0.278	0.441	0.198	−0.39	0.00114
77	69	48.0	0.330	0.280	0.385	0.12	0.00138
70	44	50.0	0.218	0.280	0.385	−1.26	0.00185
50	59	64.8	0.265	0.280	0.385	1.37	0.00320
75	33	50.2	0.596	0.280	0.385	2.77	0.00391
84	61	53.5	0.560	0.280	0.385	5.54	0.00375
84	61	53.6	0.566	0.280	0.385	5.54	0.00379
80	35	60.5	0.296	0.280	0.385	−11.47	0.00326
72	46	50.9	0.430	0.212	0.550	−0.32	0.00266
30	31	38.8	0.341	0.212	0.550	0.01	0.00230
69	44	53.9	0.441	0.212	0.550	−0.80	0.00339
54	57	55.3	0.408	0.212	0.550	0.97	0.00283
28	61	51.8	0.440	0.212	0.550	0.39	0.00212
28	61	51.0	0.453	0.212	0.550	0.39	0.00218
75	16	64.1	0.480	0.212	0.550	−11.15	0.00555
23	6	60.7	0.631	0.212	0.550	−2.30	0.00491
23	6	61.9	0.554	0.212	0.550	−2.30	0.00431
77	35	55.9	0.603	0.212	0.550	3.61	0.00448
78	45	48.3	0.601	0.212	0.550	2.87	0.00308
78	45	47.2	0.610	0.212	0.550	2.87	0.00313
34	23	41.7	0.304	0.475	0.300	−0.16	0.00152
37	72	41.0	0.235	0.475	0.300	−2.76	0.00102
84	27	54.5	0.330	0.475	0.300	0.30	0.00246
70	53	43.5	0.593	0.475	0.300	0.64	0.00276
106	88	67.3	0.373	0.475	0.300	1.39	0.00190
86	14	59.3	1.481	0.150	0.680	−4.88	0.00918
86	14	59.2	1.481	0.150	0.680	−4.88	0.00918
85	24	57.9	1.233	0.150	0.680	−2.74	0.00757
30	36	61.0	0.626	0.150	0.680	1.45	0.00313
71	24	55.0	1.462	0.150	0.680	−2.00	0.00719
71	24	50.0	1.507	0.150	0.680	−2.00	0.00741
78	14	61.6	1.012	0.150	0.680	−8.93	0.00720
78	14	63.8	0.948	0.150	0.680	−8.93	0.00674
44	45	43.6	0.501	0.150	0.680	0.24	0.00200
44	45	44.9	0.505	0.150	0.680	0.24	0.00202
72	15	65.5	0.551	0.150	0.680	−0.77	0.00359
72	15	64.9	0.539	0.150	0.680	−0.77	0.00351
54	16	67.8	0.593	0.150	0.680	3.03	0.00489
46	35	52.6	0.587	0.150	0.680	0.62	0.00250

of determining the noticeability of color differences. Since the equality of luminance is automatically maintained by the instrument, only one adjustment need be made by the observer in order to establish the match. The analysis of the results of observations is simple and unambiguous, since variations of only one quantity need be considered. Fifty adjustments of the instrument for color match were customarily made at one sitting. The average of the observed angles was taken as the correct reading for the color match. This practice compensated automatically for any otherwise undetectable instrumental errors. The difference between the average and the angle to which the other prism was set throughout the series was an indication of the perfection with which the instrument had been prepared for the observations. In cases of appreciable discrepancies, the observations were repeated after the instrumental arrangements had been improved. Such imperfect arrangements of the instrument arose principally from slight nonuniformities in the filters. The experimenter customarily made a short series of observations of his own, and usually detected any serious disagreement between the angles of the two prisms at color match, and corrected the cause of such discrepancy before the regular observer commenced his observations.

Each of the angles observed was then subtracted from the average for the series, and these deviations were squared. The sum of the squares of the deviations was divided by the number of observations less one. The square root of this quantity is the standard deviation for a single observation of the angle for color match. This deviation of prism angle can be converted to standard deviation of distance in the chromaticity diagram by the use of Eq. (11). This standard deviation of chromaticity match can be assigned to the chromaticity corresponding to the average observed angle.

The criterion of color match is so familiar, and with the present instrument the adjustments required to establish a color match are so easy that the method has been found to be most useful in the investigation of the noticeability of color differences. This method has proved practical in extensive tests of all kinds of color differences for a few observers, and in tests for a few important kinds of color differences, using a large group of untrained and previously unexperienced observers. Extensive tests have indicated that the just noticeable differences of

Table III.—Continued.

0° Filter number	90° Filter number	θ° Average setting	$\Delta\theta$° Standard deviation	Average chromaticity x	Average chromaticity y	$\Delta y/\Delta x$	Δs Standard deviation
106	34	59.0	0.464	0.510	0.236	0.53	0.00234
39	70	44.0	0.453	0.510	0.236	−0.08	0.00173
25	53	52.9	0.400	0.510	0.236	1.30	0.00202
37	91	45.1	0.356	0.510	0.236	−5.87	0.00114
38	81	50.9	0.452	0.510	0.236	−0.91	0.00120
65	52	49.7	0.428	0.510	0.236	2.08	0.00185
65	52	49.6	0.327	0.510	0.236	2.08	0.00141
65	52	49.4	0.405	0.510	0.236	2.08	0.00173
69	33	48.8	0.643	0.380	0.498	−11.90	0.00264
95	57	51.7	0.329	0.380	0.498	−3.02	0.00191
23	59	62.1	0.760	0.380	0.498	2.03	0.00373
23	59	60.8	0.939	0.380	0.498	2.03	0.00462
99	32	63.1	0.560	0.380	0.498	7.19	0.00353
68	110	54.9	0.388	0.380	0.498	−1.09	0.00132
68	110	51.9	0.444	0.380	0.498	−1.09	0.00150
108	31	40.4	0.655	0.380	0.498	0.52	0.00163
108	31	41.8	0.833	0.380	0.498	0.52	0.00208
60	109	62.8	0.848	0.380	0.498	1.19	0.00219
60	109	62.4	0.894	0.380	0.498	1.19	0.00224
64	79	30.4	0.277	0.160	0.200	0.02	0.00099
29	75	45.7	0.390	0.160	0.200	0.65	0.00095
82	8	55.2	0.376	0.160	0.200	−1.44	0.00160
47	85	62.2	0.227	0.160	0.200	4.53	0.00168
66	102	62.6	0.353	0.160	0.200	2.46	0.00128
74	112	41.6	1.127	0.160	0.200	−6.06	0.00203
84	104	47.5	0.270	0.390	0.237	0.03	0.00126
80	87	48.6	0.459	0.390	0.237	0.82	0.00240
80	87	48.4	0.452	0.390	0.237	0.82	0.00236
25	93	44.7	0.248	0.390	0.237	−1.76	0.00098
42	69	60.5	0.225	0.390	0.237	92.3	0.00123
92	91	63.4	0.258	0.390	0.237	2.42	0.00178
26	89	66.8	0.258	0.390	0.237	1.64	0.00216
37	98	50.9	0.176	0.390	0.237	−0.54	0.00107
65	99	63.2	0.286	0.390	0.237	−1.38	0.00105
100	27	48.2	0.274	0.390	0.237	0.42	0.00197
100	27	46.9	0.237	0.390	0.237	0.42	0.00170
70	72	65.1	0.471	0.385	0.393	−26.10	0.00242
79	109	66.8	0.440	0.385	0.393	5.15	0.00306
97	67	58.1	0.511	0.385	0.393	1.74	0.00361
97	67	59.0	0.506	0.385	0.393	1.74	0.00357
99	31	48.1	0.771	0.385	0.393	2.61	0.00375
103	94	59.3	0.204	0.385	0.393	−0.67	0.00160
98	51	44.9	0.277	0.385	0.393	0.39	0.00210
23	62	44.5	0.552	0.385	0.393	0.85	0.00247
70	23	52.0	0.412	0.344	0.284	−1.01	0.00089
102	73	47.3	0.197	0.344	0.284	0.09	0.00102
84	96	48.0	0.409	0.344	0.284	0.61	0.00142
97	95	45.7	0.557	0.344	0.284	0.88	0.00190
80	69	59.9	0.393	0.344	0.284	2.70	0.00198
80	69	60.4	0.437	0.344	0.284	2.70	0.00222
79	108	45.1	0.394	0.344	0.284	−12.85	0.00163
76	93	63.9	0.310	0.270	0.275	8.94	0.00178
74	116	47.8	0.427	0.228	0.250	1.38	0.00228
76	98	63.0	0.336	0.228	0.250	−5.38	0.00142
83	93	61.5	0.507	0.228	0.250	2.84	0.00296
82	94	58.0	0.378	0.228	0.250	6.05	0.00272
79	8	47.2	0.158	0.228	0.250	−0.38	0.00080
50	23	60.1	0.316	0.228	0.250	0.94	0.00160
100	30	60.2	0.397	0.152	0.365	−2.09	0.00347
83	113	70.0	0.353	0.152	0.365	−7.23	0.00360
84	28	56.0	0.514	0.152	0.365	−1.29	0.00324
64	71	58.5	0.373	0.152	0.365	3.23	0.00265
8	94	45.4	0.416	0.152	0.365	0.89	0.00175
23	55	54.2	0.526	0.152	0.365	−0.23	0.00232
66	76	47.5	0.221	0.187	0.118	0.15	0.00054
107	75	56.5	0.652	0.187	0.118	2.88	0.00207
107	75	56.0	0.608	0.187	0.118	2.88	0.00193
107	75	52.9	0.700	0.187	0.118	2.88	0.00222
63	98	65.3	0.373	0.187	0.118	6.82	0.00197
63	98	64.5	0.324	0.187	0.118	6.82	0.00172
82	50	35.6	0.393	0.187	0.118	−0.41	0.00057
47	100	59.6	0.310	0.187	0.118	1.02	0.00092
40	101	63.3	0.236	0.187	0.118	−3.61	0.00082
26	8	45.5	0.160	0.253	0.125	−0.97	0.00057
26	101	52.2	0.158	0.253	0.125	−0.82	0.00047
107	80	66.2	0.390	0.253	0.125	0.76	0.00153
82	106	37.6	0.277	0.253	0.125	0.28	0.00067
63	96	64.6	0.292	0.253	0.125	1.22	0.00227
41	97	57.7	0.392	0.253	0.125	3.84	0.00121
40	99	62.1	0.437	0.253	0.125	1.71	0.00221
117	100	43.8	0.502	0.160	0.057	1.41	0.00084
117	100	45.3	0.458	0.160	0.057	1.41	0.00079
118	106	30.3	0.265	0.160	0.057	0.44	0.00047
4	83	56.4	0.493	0.160	0.057	2.56	0.00081
40	47	62.3	0.278	0.160	0.057	−0.32	0.00036
40	47	58.5	0.276	0.160	0.057	−0.32	0.00036
63	66	50.9	0.287	0.160	0.057	−1.30	0.00040
26	87	53.7	0.564	0.365	0.153	1.16	0.00294
26	87	52.4	0.610	0.365	0.153	1.16	0.00318
26	87	53.0	0.577	0.365	0.153	1.16	0.00302
92	81	49.3	0.576	0.365	0.153	1.67	0.00189
25	80	52.5	0.462	0.365	0.153	−0.24	0.00122
42	79	52.6	0.474	0.365	0.153	−2.43	0.00078
76	38	41.5	0.332	0.365	0.153	0.21	0.00175
119	70	51.5	0.590	0.365	0.153	−3.18	0.00132
41	105	67.6	0.607	0.365	0.153	0.66	0.00336
92	81	51.0	0.554	0.365	0.153	1.67	0.00182
41	105	67.3	0.666	0.365	0.153	0.66	0.00369
38	67	56.3	0.185	0.527	0.350	−4.08	0.00128
106	52	70.1	0.284	0.527	0.350	1.25	0.00201
48	60	44.0	0.293	0.527	0.350	−0.23	0.00189
23	27	54.9	0.332	0.527	0.350	0.07	0.00194
120	53	62.8	0.416	0.527	0.350	0.44	0.00252
79	71	54.5	0.185	0.305	0.323	−2.59	0.00101
75	17	43.4	0.361	0.305	0.323	1.03	0.00203
74	67	56.8	0.247	0.305	0.323	1.28	0.00228
84	116	49.9	0.471	0.305	0.323	1.76	0.00213
112	95	50.9	0.191	0.305	0.323	0.27	0.00095
99	78	34.1	0.272	0.305	0.323	−0.84	0.00085
84	72	45.7	0.475	0.305	0.323	2.09	0.00215
79	48	47.1	0.389	0.596	0.283	0.34	0.00241
39	51	38.9	0.434	0.596	0.283	−5.70	0.00132
37	27	59.8	0.376	0.596	0.283	1.20	0.00152
43	69	35.9	0.324	0.596	0.283	−0.55	0.00138
21	73	50.2	0.440	0.596	0.283	−0.14	0.00203
81	49	51.0	0.503	0.596	0.283	0.03	0.00236
28	61	37.5	0.377	0.131	0.521	0.39	0.00212
94	30	55.3	0.560	0.131	0.521	−0.36	0.00222
8	36	55.7	0.366	0.131	0.521	3.60	0.00326
77	24	48.8	0.512	0.131	0.521	−7.21	0.00358
77	24	47.3	0.591	0.131	0.521	−7.21	0.00415
78	44	43.0	0.956	0.131	0.521	−2.00	0.00418
98	44	57.8	0.643	0.131	0.521	−2.04	0.00474
23	46	64.6	0.546	0.131	0.521	−1.12	0.00389
23	46	65.0	0.587	0.131	0.521	−1.12	0.00416
76	23	57.4	0.342	0.278	0.223	3.80	0.00137
76	23	57.5	0.360	0.278	0.223	3.80	0.00145
100	99	49.2	0.494	0.278	0.223	1.05	0.00129
100	99	49.1	0.518	0.278	0.223	1.05	0.00136
82	69	56.7	0.367	0.278	0.223	1.61	0.00224
79	75	51.8	0.258	0.278	0.223	−0.42	0.00046
106	75	62.3	0.197	0.278	0.223	−0.70	0.00059
82	69	55.1	0.411	0.278	0.223	1.61	0.00252
80	86	41.8	0.241	0.278	0.223	−2.91	0.00089
74	120	68.4	0.407	0.278	0.223	0.60	0.00103
74	120	69.0	0.407	0.278	0.223	0.60	0.00103
80	98	47.2	0.263	0.278	0.223	−1.89	0.00068
76	79	52.7	0.798	0.300	0.163	0.79	0.00181
26	120	60.6	0.237	0.300	0.163	158.0	0.00080
26	120	60.4	0.237	0.300	0.163	158.0	0.00080
83	105	49.7	0.255	0.300	0.163	0.33	0.00121
106	100	46.2	0.261	0.300	0.163	−0.16	0.00062
41	96	61.1	0.459	0.300	0.163	1.48	0.00283
41	96	61.3	0.413	0.300	0.163	1.48	0.00253
92	97	59.3	0.258	0.300	0.163	−2.36	0.00076
70	62	62.6	0.520	0.472	0.399	2.22	0.00290
38	109	64.3	0.204	0.472	0.399	−1.69	0.00158
103	72	56.1	0.278	0.472	0.399	−1.18	0.00155
69	52	41.7	0.449	0.472	0.399	0.14	0.00152
73	67	49.9	0.431	0.472	0.399	−16.0	0.00176
23	58	57.3	0.422	0.472	0.399	0.40	0.00231
81	17	54.3	0.456	0.472	0.399	5.85	0.00201
99	51	60.6	0.546	0.472	0.399	0.80	0.00269
87	116	45.8	0.352	0.472	0.399	−0.72	0.00111
99	51	58.9	0.593	0.472	0.399	0.80	0.00292

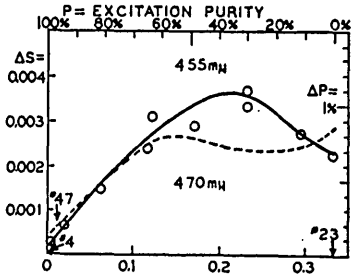


FIG. 8. Standard deviations of purity matching for dominant wave-length 455 mμ (PGN, solid curve and circles); and 470 mμ (DLM, broken curve).

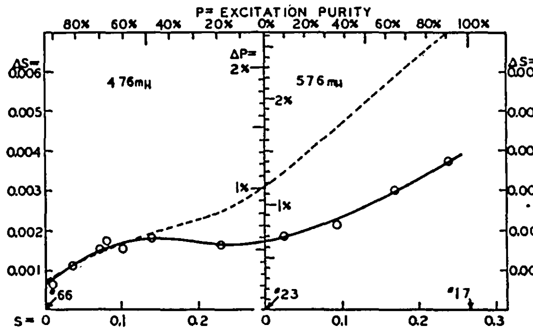


FIG. 9. Standard deviations of purity matching for dominant wave-lengths 476 and 576 mμ (PGN, solid curve and circles; DLM, broken curve).

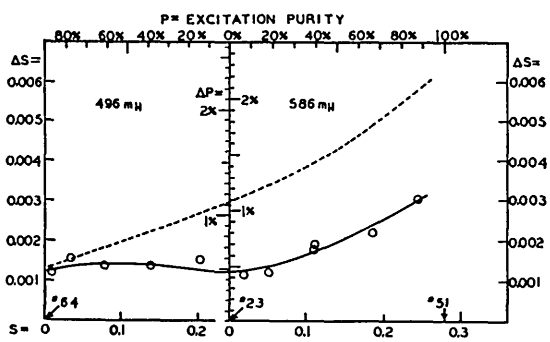


FIG. 10. Standard deviations of purity for dominant wave-lengths 496 and 586 mμ (PGN, circles and solid curve; DLM, broken curve).

color, for constant luminance of approximately 15 millilamberts, are proportional to the corresponding standard deviations of color matching by the same observer. Fluctuations of the criterion of just noticeability will, of course, alter the constant of proportionality, but this factor (approximately three) is constant throughout any series of observations of the kind specified in which the criterion of just noticeability has been proved to be constant. Therefore, the

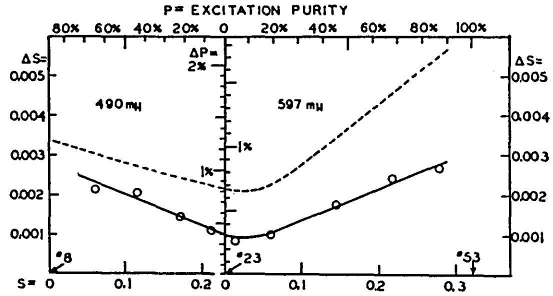


FIG. 11. Standard deviations of purity for dominant wave-lengths 490 and 597 mμ (PGN, circles and solid curve; DLM, broken curve).

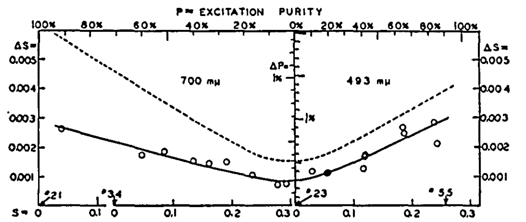


FIG. 12. Standard deviations of purity for dominant wave-lengths 493 and 700 mμ (PGN, circles and solid curve; DLM, broken curve).

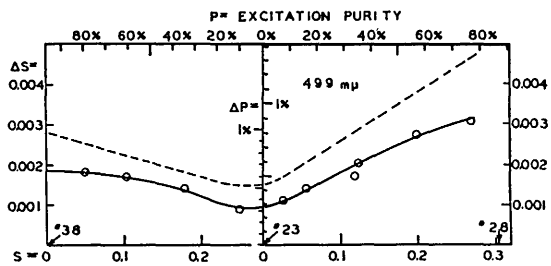


FIG. 13. Standard deviations of purity for dominant wave-length 499 mμ and complementary (PGN, circles and solid curve; DLM, broken curve).

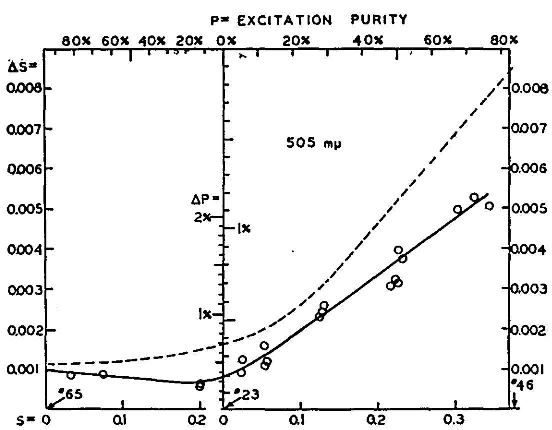


FIG. 14. Standard deviations of purity for dominant wave-length 505 mμ and complementary (PGN, circles and solid curve; DLM, broken curve).

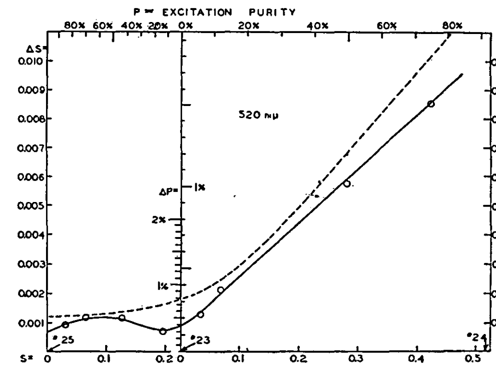


FIG. 15. Standard deviations of purity for dominant wave-length 520 mμ and complementary (PGN, circles and solid curve; DLM, broken curve).

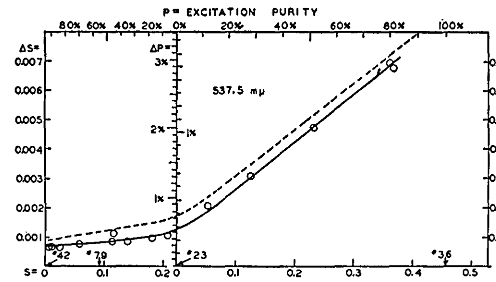


FIG. 16. Standard deviations of purity for dominant wave-length 537.5 mμ and complementary (PGN, circles and solid curve; DLM, broken curve).

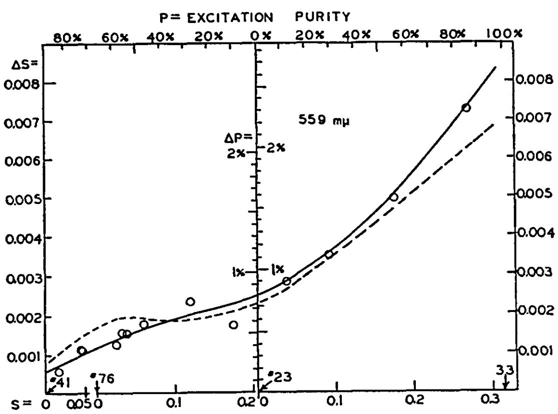


FIG. 17. Standard deviations of purity for dominant wave-length 559 mμ and complementary (PGN, circles and solid curve; DLM, broken curve).

standard deviation of color matching is considered to be a satisfactory measure of the noticeability of color differences, equivalent in significance to the just noticeable difference and more consistent in magnitude.

Tables II and III and the circled points and solid curves in Figs. 8 to 47 represent the results

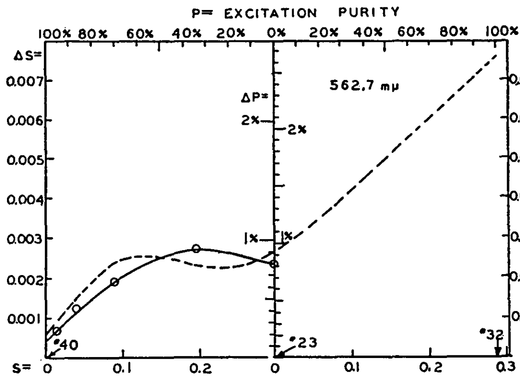


FIG. 18. Standard deviations of purity for dominant wave-length 562.7 mμ and complementary (PGN, circles and solid curve; DLM, broken curve).

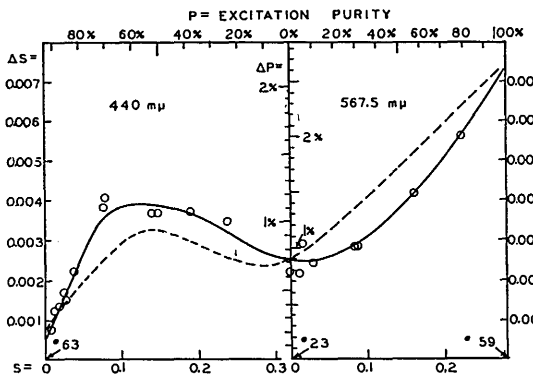


FIG. 19. Standard deviations of purity for dominant wave-lengths 440 and 567.5 mμ (PGN, circles and solid curve; DLM, broken curve).

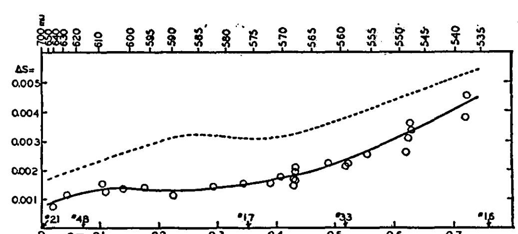


FIG. 20. Standard deviations of chromaticity along spectrum locus from 535 to 700 mμ (PGN, circles and solid curve; DLM, broken curve).

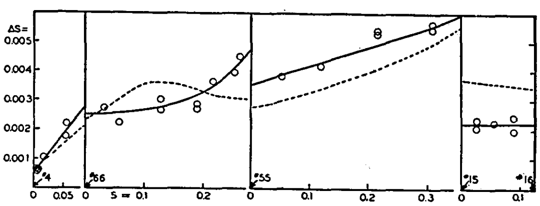


FIG. 21. Standard deviations of chromaticity along lines near spectrum locus from 400 to 535 mμ (PGN, circles and solid curves; DLM, broken curve).

derived from the observations of a single observer. Mr. Perley G. Nutting, Jr., whose interest and patience is acknowledged here with gratitude and admiration, has made nearly 25,000 settings for color match, and practically all of his results are represented in the tables and curves. Figures 8 to 19 represent the standard deviations of matching the purities of the indicated chromaticities, when purity alone was variable and subject to error. The dominant wave-lengths are indicated on the figures. Figure 20 represents the standard deviations of chromaticity matching when the chromaticity varied along a straight line very close to the spectrum locus from 533

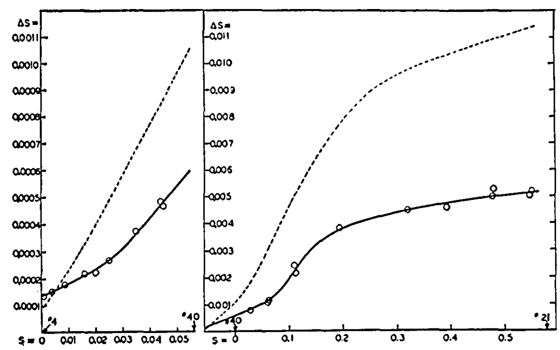


FIG. 22. Standard deviations of chromaticity along the straight line connecting points representing 440 and 700 mμ (PGN, circles and solid curve; DLM, broken curve). Note enlarged scale of portions of curve shown at left.

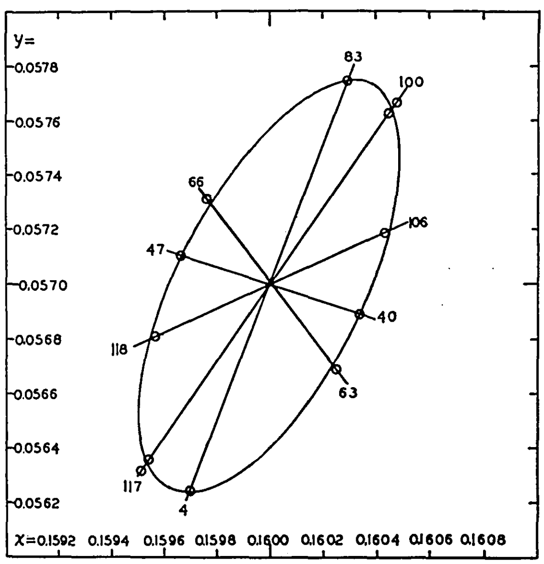
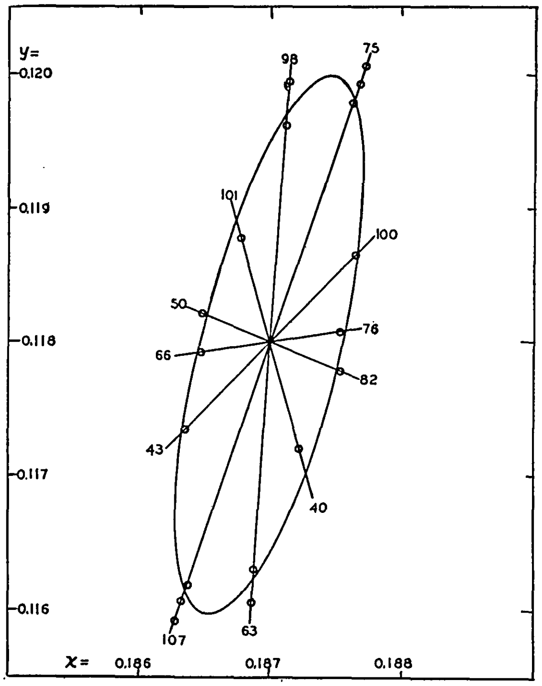


FIG. 23. Standard deviations of chromaticity from point ($x=0.160$, $y=0.057$), observer: PGN.



FIGS. 24 TO 47. Standard deviations of chromaticity from indicated standards, observer: PGN.

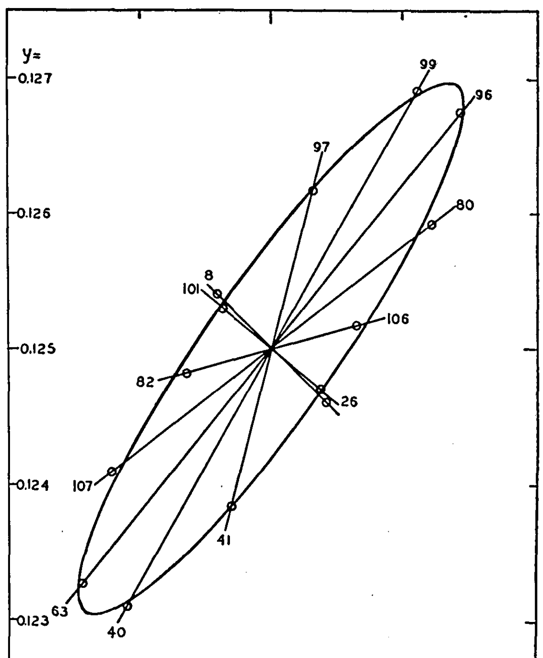


FIG. 25.

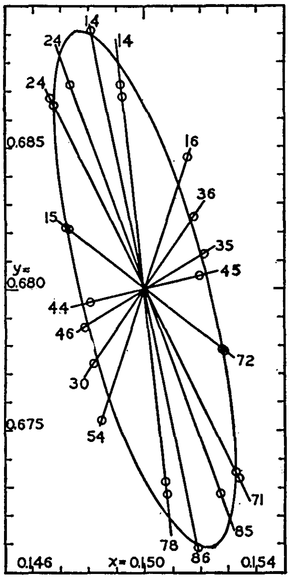


FIG. 26.

mμ to 700 mμ. Figure 21 represents the standard deviations of matching the chromaticity along straight lines close to the spectrum locus from 440 mμ to 533 mμ. Figure 22 represents the standard deviations of matching the chromaticities along the straight line from the spectrum locus at 440 mμ to the spectrum locus at 700 mμ. This last curve corresponds to the noticeability of complementary wave-length differences for highly saturated purples.

The broken curves in Figs. 8 to 22 represent the results for similar observations by the author. The discrepancies between the results for these two observers are significantly large, and appear to be correlated with position and direction of the deviations in the chromaticity diagram. Both observers have normal color vision according to all of the usual tests. The magnitude and character of the discrepancies shown in Figs. 8 to 22 are comparable with those reported by Wright.[5]

The ellipses shown in Figs. 23 to 47 represent the noticeability of chromaticity variations in all directions from the chromaticities indicated at the centers of the ellipses. The observations represented by each ellipse were obtained by selecting from five to eight pairs of filters, each pair of which could be used in the instrument to synthesize a chromaticity very closely approximating that represented by the center of the ellipse. The data represented in Figs. 23 to 47 are summarized in Table III. In every case, a filter was available which would alone produce the chromaticity indicated by the center of the ellipse. The angle to which the prisms of the

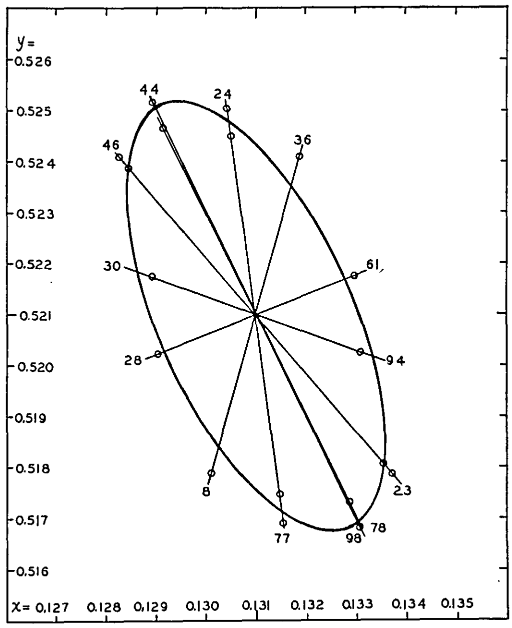


FIG. 27.

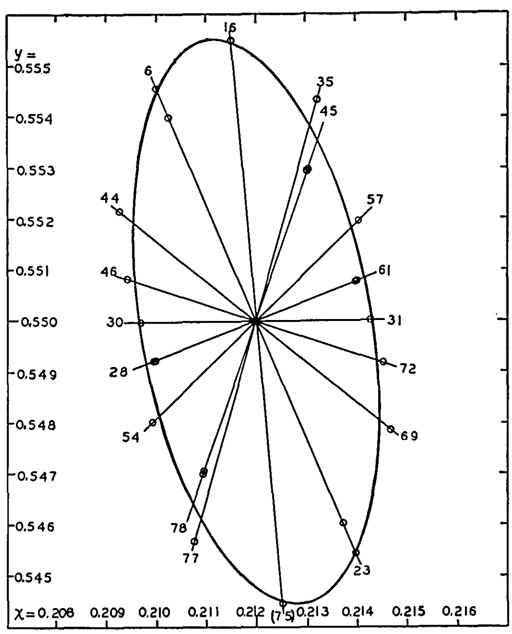


FIG. 28.

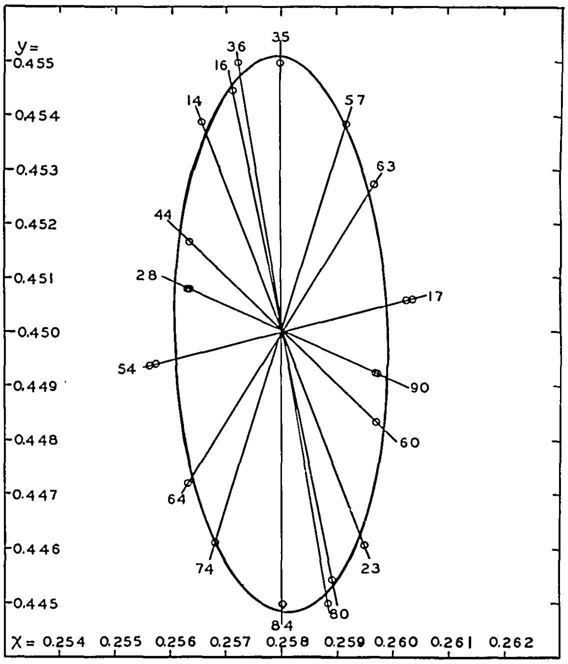


FIG. 29.

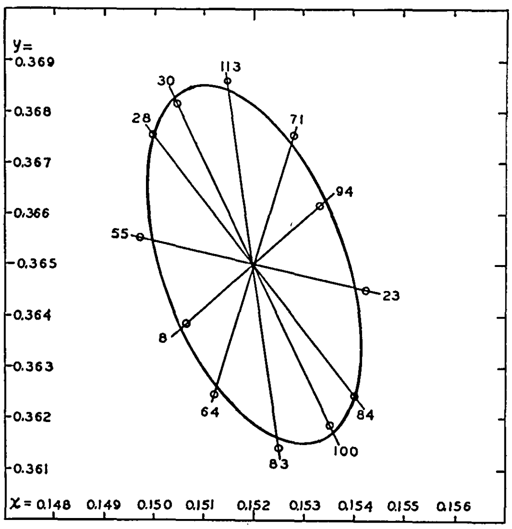


FIG. 30.

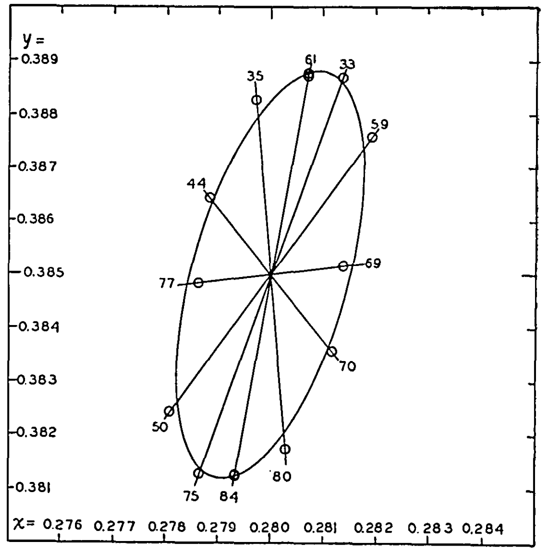


FIG. 31.

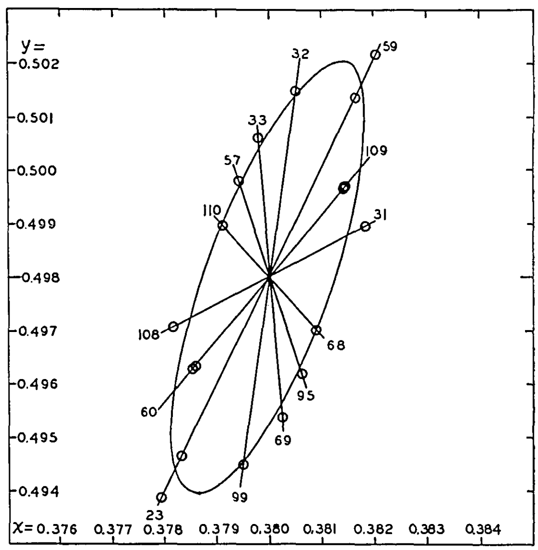


FIG. 32.

instrument should be set in order to synthesize this standard chromaticity with each pair of filters was both calculated and confirmed by observation of the resultant mixture in comparison with the light transmitted by the single filter. With one of the prisms set at this angle, so as to synthesize the desired chromaticity, the other prism was rotated by the observer until a satisfactory match was obtained. The standard deviation of fifty observations was determined. This deviation was represented by two opposite radii from the center point towards the points representing the chromaticities of the two filters used for the synthesis. Each pair of filters used for the synthesis of the standard color resulted in the establishment of another diameter of the ellipse. The complete ellipse is drawn through the points at the ends of these diameters. Several different coordinate scales are employed in

Figs. 23 to 47 in order to represent the ellipses with an accuracy comparable to that of the observations. All of these ellipses are represented to the same scale on the composite diagram in Fig. 48. In this figure, every ellipse is drawn ten times its correct size with relation to the coordinate scale of the chromaticity diagram. The centers of the ellipses are placed at their proper locations in the chromaticity diagram.

Sufficient ellipses are shown in Fig. 48 to make possible the estimation of the shape and size of the ellipse around the point representing any chromaticity ordinarily encountered in art or commerce. From such an ellipse the noticeability of a color difference in any direction can be deduced. For instance, the ordinates of the curves shown in Figs. 8 to 22 could be determined from a sufficiently complete series of ellipses. Such a series of ellipses would therefore represent all the information contained in all the curves for the noticeabilities of purity and dominant wave-length change. In addition, these

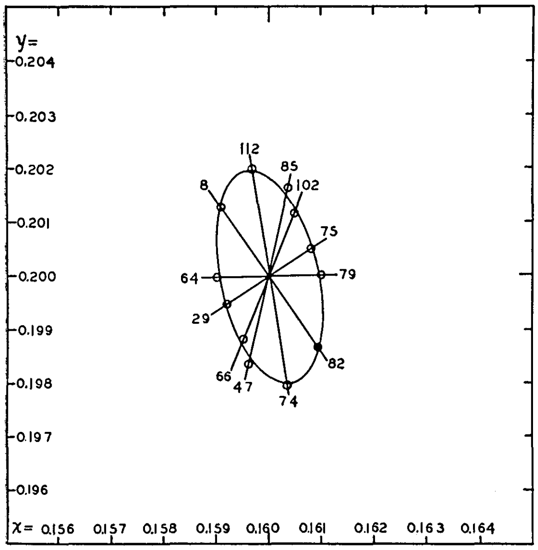


FIG. 33.

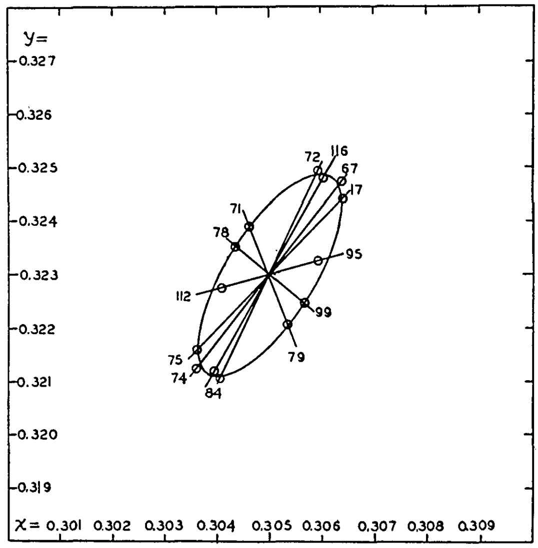


FIG. 35.

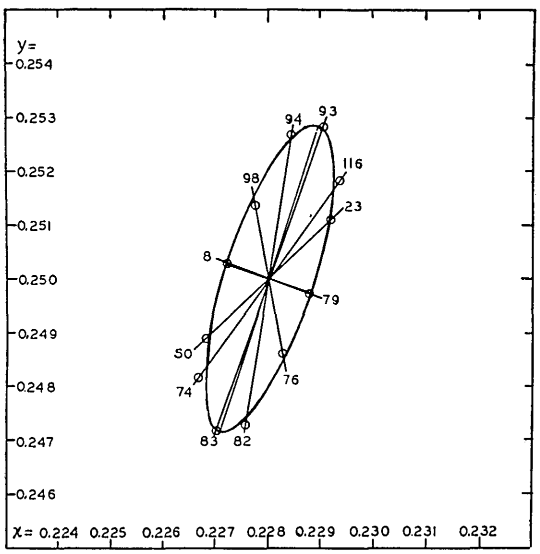


FIG. 34.

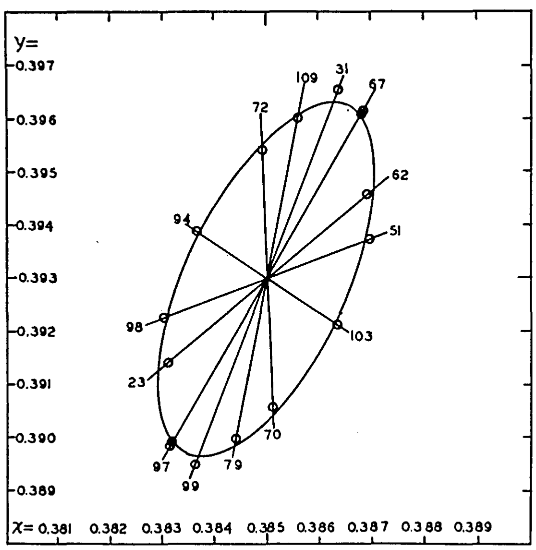


FIG. 36.

ellipses represent the noticeabilities of all conceivable combinations of purity and dominant wave-length differences.

Figure 49 represents in terms of wave-length the standard deviations of color matching for spectral colors, at constant luminance. This curve was derived from Figs. 20 and 21, together with the information contained in Fig. 48, which permits the estimation of noticeabilities of color differences in other directions, and for other chromaticities than those actually observed. Undue emphasis should not be placed on this curve, since it is based on extrapolations of observed data. This curve facilitates comparison of the present results with previous results for wave-length discrimination. Such data represent a limiting special case of color discrimination. The curve in Fig. 49 cannot be expected to be as reliable as the best results secured with apparatus and methods designed specifically for the investigation of wave-length discrimination in the spectrum. Conversely, the data for

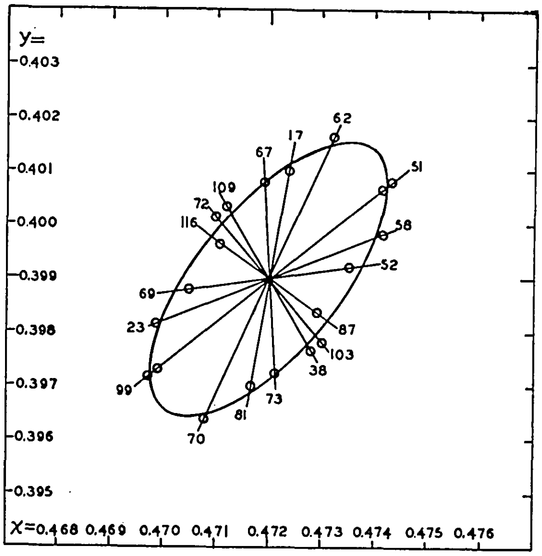


FIG. 37.

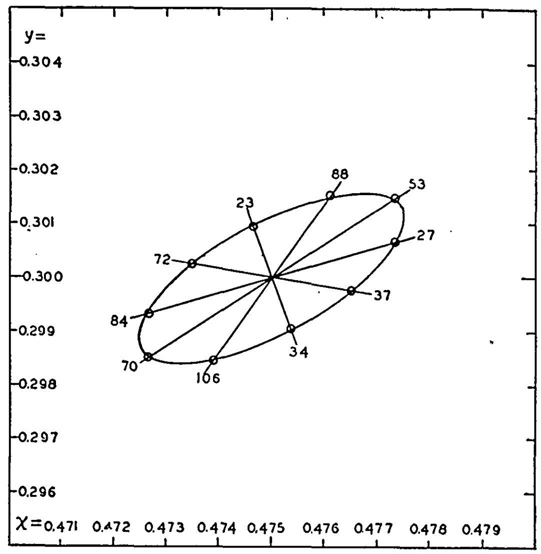


FIG. 39.

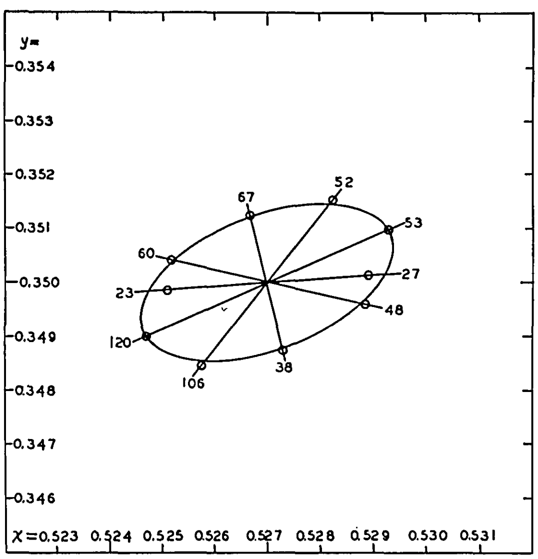


FIG. 38.

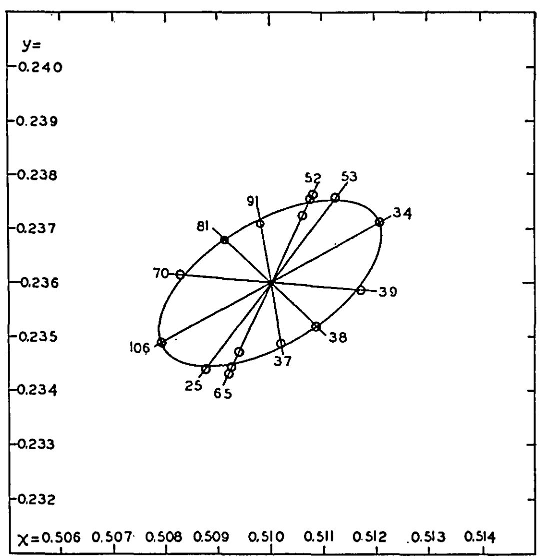


FIG. 40.

discrimination of colors of less than spectral purity, represented in Figs. 8 to 47, inclusive, secured with an instrument designed specifically for their investigation, should be more reliable than similar results obtained with apparatus primarily designed for investigations of color matching and discrimination in the pure spectrum. The curve in Fig. 49 is essentially in agreement with the results of König and Dieterici[10] and of Wright and Pitt.[12] The portion of the curve from 550 mμ to 700 mμ, which shows no evidence of a secondary minimum near 630 mμ frequently reported,[13–16] is derived directly,

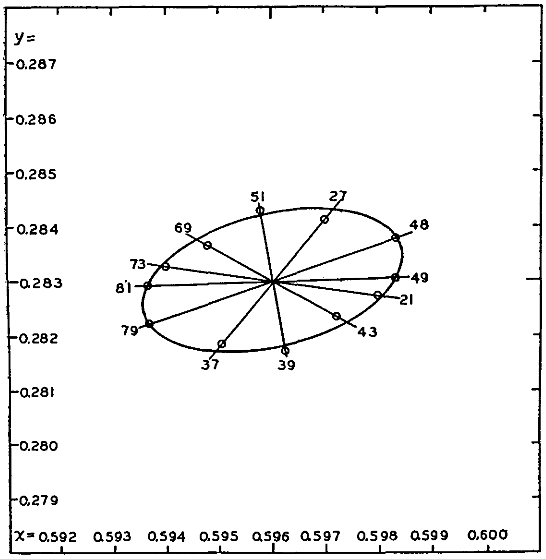


FIG. 41.

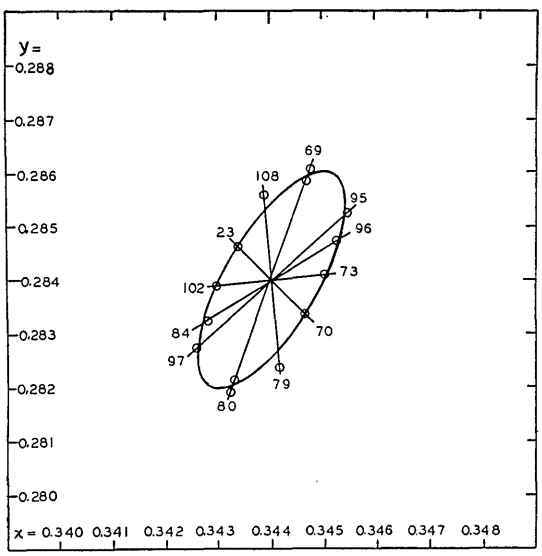


FIG. 42.

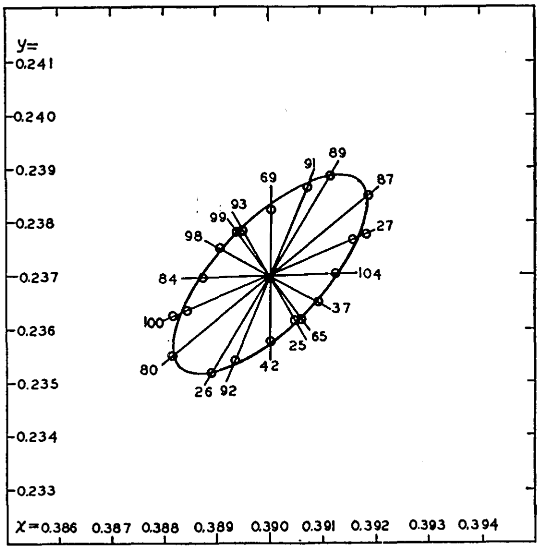


FIG. 43.

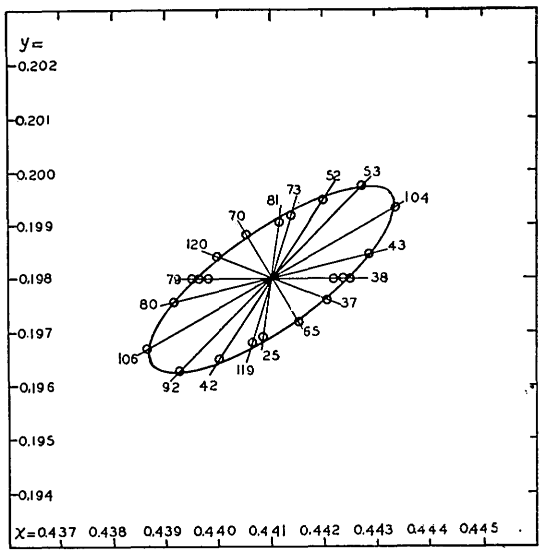


FIG. 44.

[12] W. D. Wright and F. H. G. Pitt, "Hue discrimination in normal colour-vision," Proc. Phys. Soc. **46**, 459–468 (1934).

[13] O. Steindler, "Die Farbenempfindlichkeit des normalen und farbenblinden Auges," Sitzungsber. Akad. Wiss. Wien, Math.-Naturw. Kl., Abt. 2a, **115**, 39 (1906).

[14] L. A. Jones, "The fundamental scale for pure hue and retinal sensibility to hue differences," J. Opt. Soc. Am. **1**, 63–77 (1917).

[15] H. Laurens and W. F. Hamilton, "The sensibility of the eye to differences in wave-length," Am. J. Physiol. **65**, 547 (1923).

[16] E. P. T. Tyndall, "Chromaticity sensibility to wave-length difference as a function of purity," J. Opt. Soc. Am. **23**, 15–24 (1933).

without extrapolation, from the observed data represented in Fig. 20. The portion of the curve most dependent on extrapolation falls between 430 mμ and 470 mμ. For this reason, very little significance can be attached to the absence of the secondary minimum near 440 mμ reported by all previous investigators. Figure 50, representing the standard deviation of determination of complementary wave-lengths for highly saturated purples, has been derived directly from Fig. 22 and should be quite reliable.

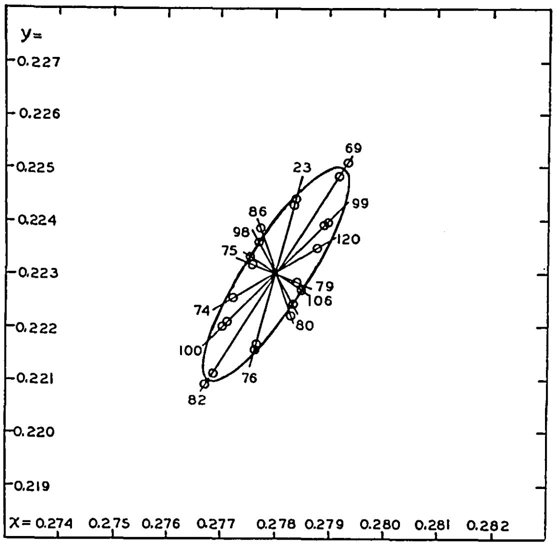


FIG. 45.

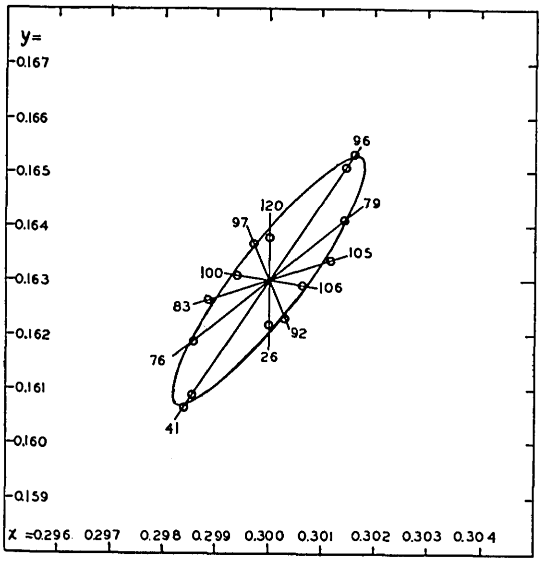


FIG. 46.

The standard deviations Δs encountered in matching a neutral color very similar to Illuminant *C* are indicated in Fig. 35, and are tabulated in the second column of Table IV. The corresponding values of excitation purity p_e were computed by dividing the values of Δs by the distance from the neutral point to the spectrum locus for each wave-length. Colorimetric purity p_c was computed by multiplying the excitation purity by the values of a given in Table I of reference 17. The logarithms of these values of colorimetric purity are compared in Table V with similar data for eight other observers. In Table V, values of log $[p_c(570\ \text{m}\mu)/p_c(\lambda)]$ are given for these nine observers. The values of IGP and FGB were computed from the values of p_c given in Table I of reference 4. The values for FLW, WJM, and LCM were computed from the values of p_c for light-adapted eyes, given in Table V of reference 18. The data for FHGP and WDW were computed by subtracting the values for 570 mμ from the values of the logarithms given in reference 19. The values for JHN were

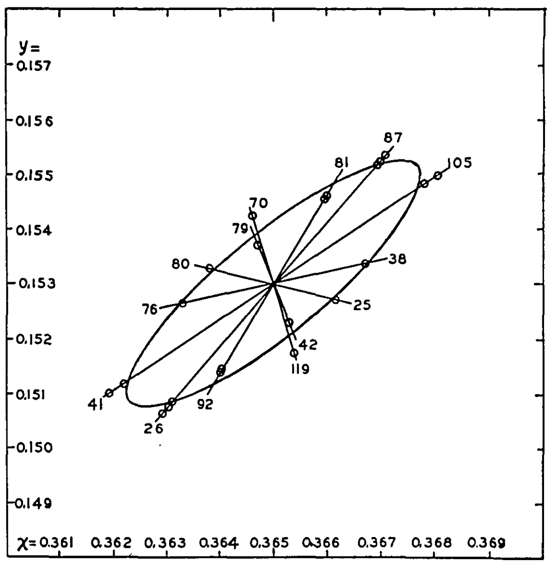


FIG. 47.

[17] D. L. MacAdam, "Photometric relationships between complementary colors," J. Opt. Soc. Am. **28**, 103–111 (1938).

[18] L. C. Martin, F. L. Warburton and W. J. Morgan, "Determination of the sensitiveness of the eye to differences in the saturation of colours," Med. Research Council, Special Report Series, No. 188 (H. M. Stationery Office, London, 1933).

[19] W. D. Wright and F. H. G. Pitt, "The saturation discrimination of two trichromats," Proc. Phys. Soc. **49**, 329–331 (1937).

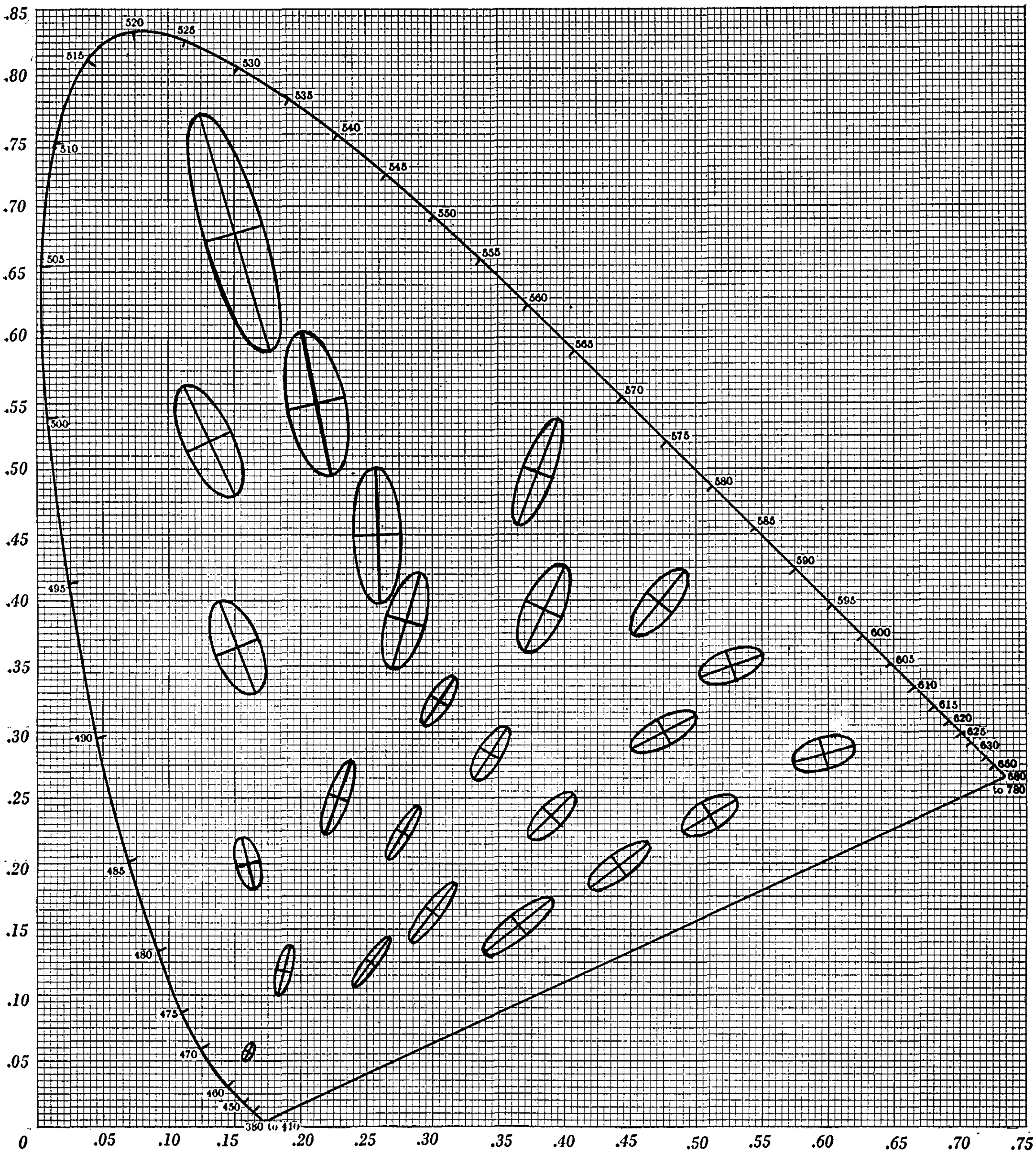


FIG. 48. Standard deviations of chromaticity from indicated standards, represented ten times actual scale on I.C.I. 1931 standard chromaticity diagram, observer: PGN.

similarly computed from those given in reference 20.

The values for PGN, computed from Table IV, are quite similar to those for the other eight observers, and to the average values, which correspond to the geometric means of the thresholds of colorimetric purity. Table IV indicates that the tremendous variation of the threshold values of colorimetric purity through the spectrum is to a great extent an artifact resulting from the use of the scale of colorimetric purity. The ellipse in Fig. 35 is considered a more satis-

[20] J. H. Nelson, "The colour vision characteristics of a trichromat," Part 2, Proc. Phys. Soc. 49, 332–337 (1937).

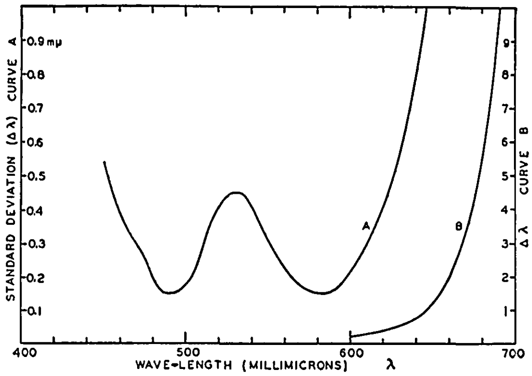


FIG. 49. Standard deviations of wave-length matching in spectrum, derived by extrapolations from Figs. 20, 21, 48.

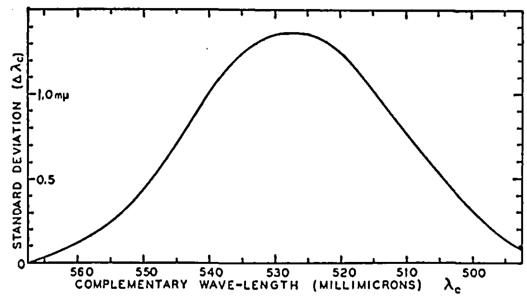


FIG. 50. Standard deviations of complementary wave-length matching for highly saturated non-spectral colors (purples).

factory representation of the data than the customary curves showing the variation of log p_c as a function of wave-length. This is one of many clear examples of the advantages secured by the use of the chromaticity diagram as the basis for the representation of color discrimination.[21] The probability of accidental misrepresentation or misinterpretation of data is less when the results are represented on a basis resulting in simple, regular figures than when they are represented by curves exhibiting marked changes of slope, and several inflections.

The representation of the noticeabilities of color differences by loci around each point in the chromaticity diagram has been suggested by several authors, including Martin, Warburton, and Morgan,[20] Judd,[22] and Silberstein,[23] but very few data of this kind have been available. The solution of the general problem of the noticeability of color difference is represented most clearly in this way. The noticeabilities of color differences involving simultaneous luminance and chromaticity variations are expected to define ellipsoids around the points representing every color in the color solid. Such ellipsoids will be required for the complete representation of color differences. The ellipses shown in Fig. 48 may be regarded as the cross sections of the ellipsoids, corresponding to constant luminance.

The development of a more elegant manner of specifying the noticeabilities of color differences than the system of ellipses suggested by Fig. 48 requires an application of the principles of differential geometry.[23] Such an analysis of the results represented in Tables II and III and Figs. 8 to 48 is now in preparation. A preliminary investigation[24] demonstrated that results of the

TABLE IV. First step from white.

Wave-length	Δs	$p_e(\%)$	$p_c(\%)$	$-\log p_c$
400	0.00205	0.59	0.0089	4.05
440	0.00209	0.62	0.0211	3.68
450	0.00218	0.65	0.0364	3.44
460	0.00219	0.66	0.047	3.33
470	0.00221	0.69	0.126	2.90
480	0.00182	0.64	0.269	2.57
490	0.00104	0.40	0.37	2.43
500	0.00086	0.234	0.40	2.40
510	0.00092	0.178	0.42	2.37
520	0.00096	0.171	0.45	2.34
530	0.00107	0.211	0.54	2.27
540	0.00116	0.265	0.63	2.20
550	0.00135	0.365	0.80	2.10
560	0.00172	0.55	1.03	1.99
565	0.00200	0.70	1.30	1.88
570	0.00223	0.82	1.41	1.85
575	0.00208	0.79	1.30	1.89
580	0.00172	0.65	1.00	2.00
585	0.00145	0.53	0.76	2.12
590	0.00126	0.435	0.58	2.24
600	0.00107	0.328	0.39	2.41
610	0.00099	0.274	0.29	2.54
620	0.00096	0.248	0.242	2.62
630	0.00095	0.233	0.215	2.67
640	0.00094	0.225	0.200	2.70
650	0.00094	0.221	0.191	2.72
660	0.00093	0.218	0.186	2.73
670	0.00093	0.216	0.183	2.74
680	0.00093	0.215	0.181	2.74
700	0.00093	0.214	0.179	2.75

[21] T. Smith, "The colour triangle and colour discrimination," *Discussion on Vision* (Physical Society, London, 1932), pp. 212–226.

[22] D. B. Judd, "Estimation of chromaticity differences and nearest color temperature on the standard 1931 I.C.I. colorimetric coordinate system," J. Opt. Soc. Am. **26** 421–426 (1936).

[23] L. Silberstein, "Investigations on the intrinsic properties of the color domain," J. Opt. Soc. Am. **28**, 63–85 (1938).

[24] D. L. MacAdam, "Projective transformations of color-mixture diagrams," J. Opt. Soc. Am. **32**, 2–6 (1942).

TABLE V. Comparisons of first steps from white for nine observers. log $[p_c(570\ m\mu)/p_c(\lambda)]$.

Wave-length	Observers IGP	FGB	FLW	WJM	LCM	FHGP	WDW	JHN	PGN	Aver-age
400 mμ	1.75	1.54							2.20	1.83
440	1.66	1.16	1.93		1.81				1.83	1.67
450	1.57								1.59	1.58
460	1.40	1.01	1.49	1.10	1.61				1.48	1.34
470	1.14	0.99				1.22	0.90		1.05	1.06
480	0.97	0.97	0.93	0.49	0.95	1.00	0.74	0.65	0.72	0.82
490	0.84	0.89	0.79	0.55	0.96	0.85	0.64	0.60	0.58	0.75
500	0.71	0.76	0.71	0.40		0.71	0.57	0.56	0.55	0.62
510	0.60	0.63	0.61	0.37	0.98	0.59	0.51	0.52	0.52	0.59
520	0.49	0.52	0.56	0.38	0.81	0.50	0.46	0.48	0.49	0.52
530	0.39	0.47	0.54	0.22	0.65	0.43	0.40	0.44	0.42	0.44
540	0.31	0.46	0.43	0.20	0.37	0.38	0.34	0.37	0.35	0.35
550	0.28	0.42				0.31	0.25	0.30	0.25	0.30
560	0.18	0.13				0.18	0.14	0.17	0.14	0.16
565	0.06	0.01	0.13	−0.18*	0.05				0.03	0.06*
570	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
575	0.03	0.22							0.04	0.09
580	0.36	0.32		0.44	0.39	0.37	0.18	0.28	0.15	0.31
585	0.46	0.39	0.26						0.27	0.34
590	0.52	0.46				0.54	0.44	0.45	0.39	0.46
600	0.63	0.58	0.50	0.39	0.77	0.63	0.63	0.55	0.56	0.58
610	0.72	0.70				0.70	0.77	0.67	0.69	0.70
620	0.82	0.78	0.71	0.44	1.05	0.74	0.85	0.76	0.77	0.77
630	0.90	0.80				0.79	0.93	0.85	0.82	0.84
640	0.98	0.82	0.78	0.55	1.12	0.81	0.99	0.96	0.85	0.87
650	1.03	0.84	0.85	0.52	1.11	0.85	1.04	1.00	0.87	0.90
660	1.06	0.85						1.06	0.88	0.96
670	1.08	0.85						1.14	0.89	0.98
680	1.08	0.86	0.92	0.85	1.29			1.18	0.89	1.01
700								1.27	0.90	1.08
Mean deviation from average	0.06	0.10	0.08	0.22	0.20	0.05	0.07	0.07	0.07	

* The value shown for WJM at 565 mμ, −0.18, is go greatly at variance with the values for all other observers that it has been omitted in computing the tabulated average. Better agreements with the averages could be secured by adding 0.24 to all of the values tabulated for WJM, and by subtracting 0.20 from all of the values tabulated for LCM. The data for these two observers would then appear to differ seriously from those of the other observers only at the wave-lengths 570 mμ and 580 mμ. If the adjustments described were made, the mean deviations from the average values would be reduced to 0.09 for both WJM and LCM.

kinds represented in Figs. 8 to 22 cannot be represented adequately by equal distances in any projective transformation of the standard 1931 I.C.I. diagram. This fact eliminates the possibility that such a transformation of the standard chromaticity diagram can constitute a uniform chromaticity-scale diagram[3] adequately representing the present data. The more general representations[23, 25, 26] which remain as possibilities for the construction of a truly uniform chromaticity-scale diagram have very complicated geometrical characteristics. The standard I.C.I. chromaticity diagram seems preferable for practical use, even though the system of ellipses shown in Fig. 48 is required as supplementary data. The standard chromaticity diagram appears as convenient as possible for the representation of color differences,[21] in the same manner that plane maps of large portions of the surface of the earth are most convenient, regardless of the distortions of distances introduced by such representations.

[25] R. H. Sinden, "A further search for the ideal color system," I. "A new mechanico-graphical method," J. Opt. Soc. Am. **27**, 124–131 (1937); II. "A reconsideration of the Helmholtz line element," J. Opt. Soc. Am. **28**, 339–347 (1938).

[26] W. Peddie, "The general applicability of Fechner's law in colour sensation," Nature **124**, 791–792 (1929); "Colour vision and chromaticity scales," Nature **146**, 717–718 (1940).

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The author gratefully acknowledges the interest and assistance of Dr. Loyd A. Jones and Mr. Earl M. Lowry of these Laboratories, who suggested this investigation, made available the special equipment employed, and contributed essential ideas and advice at all stages of the work. The highly satisfactory performance of the instrument is due in large measure to Mr. Ralph E. McAdam who was responsible for the mechanical details of its design, and to Mr. Max Zill who constructed the instrument with great care and skill. It is a pleasure to acknowledge again the essential contributions of Mr. Perley G. Nutting, Jr., whose critical interest in the ideas involved in this research was much greater and more helpful than is implied by the fact that he was the principal observer. Many persons have made more or less extensive series of observations, all of which contributed to the development of the method finally adopted and to the understanding of the complex problems involved.

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Calculation of Illumination from Triangular Sources

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(Received January 4, 1942)

1. INTRODUCTION

ALTHOUGH actual light sources with triangular boundaries occur but infrequently in practice, an important class of problems can be solved by the methods introduced in this paper. A knowledge of formulae for the triangular source allows the computer to calculate the luminous flux density, at any point on any surface, produced by a source having rectilinear outlines. Any such source can be decomposed into triangles and the flux densities produced by the individual triangles can be computed.

A special case of frequent occurrence is the rectangular window, only part of which is effective in illuminating a given surface. The plane of a tilted surface such as a drafting board will ordinarily cut the rectangular window into two parts, only one of which is useful in illuminating the board. Either the shape of this illuminating source is itself triangular or it may be decomposed into two, three, or four triangles. The problem is an important one in illuminating engineering and it is strange that no adequate solution has hitherto been available.

The paper presents a derivation of general equations for the flux density from triangular windows. A set of nomographs for the convenient solution of these equations is then given and an illustrative example is worked out.

2. CHOICE OF THE COORDINATE SYSTEM

The use of the flux-density vector **D** is convenient in this work.[1] A Cartesian coordinate system is selected with the point P (Fig. 1) as origin. The flux densities on three mutually perpendicular planes through P are called D_x, D_y, and D_z and are considered as components of the vector **D**.

The flux density at P on a tilted surface having the normal vector N is obtained by vector addi-

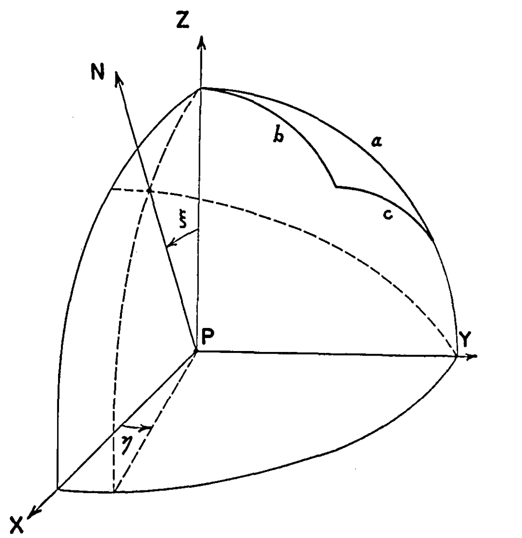

FIG. 1. The coordinate system and the spherical triangle.

[1] See, for example, A. Gershun, "The light field," translated by Moon and Timoshenko, J. Math. Phys. **18**, 51 (1939).